The singing vortex

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Marine propellers display several forms of cavitation. Of these, propeller-tip vortex cavitation is one of the important factors in propeller design. The dynamic behaviour of the tip vortex is responsible for hull vibration and noise. Thus, cavitation in the vortices trailing from tips of propeller blades has been studied extensively. Under certain circumstances cavitating vortices have been observed to have wave-like disturbances on the surfaces of vapour cores. Intense sound at discrete frequencies can result from a coupling between tip vortex disturbances and oscillating sheet cavitation on the surfaces of the propeller blades. This research article focuses on the dynamics of vortex cavitation and more in particular on the energy and frequency content of the radiated pressures.

1. Introduction

Euler, in his [1] memoir on the theory of hydraulic machines, was probably the first person to conjecture that cavitation might be a problem. As early as 1873, Reynolds [2] had carried out a fundamental study of cavitation in tubular constrictions. However, the real impetus for research in this area came from the introduction of the marine propeller. In fact, R. E. Froude, a naval architect, coined the term ‘cavitation’ ca 1895.

The conjecture of Froude proved true during the initial trials of Parson’s high powered boat ‘Turbinia’ in early 1895, shown in figure 1. The boat reached a disappointing speed of only just below 20 kts, whereas the boat was designed for a top speed well over 30 kts [3]. Parsons is likely to be the first person who was confronted with one of the problems of cavitation: thrust breakdown on a marine propeller. His approach to solving this cavitation problem sets the stage for the techniques used in modern research. The HMS Turbinia was designed by him to demonstrate the application of the compound steam turbine in marine propulsion. As already noted, the first trials were miserable failures. Parsons examined this problem in a water tunnel with a special stroboscope, both of his own design. His water tunnel is probably the world’s first cavitation tunnel to observe the cavitation phenomenon. He determined that the heavily loaded single screw on the vessel was cavitating and was unable to produce the required thrust. His solution was to replace the original propulsion system with three smaller turbines with the same total power driving nine propellers. A speed of 32.6 kts was finally achieved, amply demonstrating the application of steam-turbine technology in marine propulsion. This brought fame and fortune to Parsons and his colleagues.

A historical overview of measurements in cavitating flows is given in the Handbook of fluid mechanics measurements [4]:

Other noteworthy efforts include the work of Barnaby and Thornycroft (1898) [5], who were studying problems that evolved during the trials of the HMS Daring. This early work led to the classical study of the potency for damage due to bubble collapse by Lord Rayleigh (1917) [6]. Minnaert 1933 [7] then set the stage for understanding the acoustics of cavitation and gas filled bubbles. The foundations of our knowledge of bubble dynamics and its interrelationship to cavitation are found in several papers by Plesset and his co-workers (Plesset 1949 [8], Plesset and Zwick 1954 [9], Epstein and Plesset 1950 [10], Plesset 1957 [11]).

The objective of the current paper is to investigate the character of radiated noise of a cavitating vortex, such as present on almost all propeller-driven
2. Cavitation on marine propellers

2.1. Types of cavitation

Cavitation may occur in a variety of forms in the flow over a marine propeller. A review of the most frequent forms of cavitation is given in figure 2. Different forms of cavitation may lead to different forms of problems. Today, most of the cavitation hindrance occurs through hull pressure fluctuations which may set parts of the ship into vibration, in particular when the excitation frequency coincides with a natural frequency of the construction in the vicinity of the propeller. The types of cavitation responsible for vibration problems are sheet cavity dynamics and tip-vortex cavity dynamics, in particular in the region that is closest to the hull plating above the propeller.

Another important source of cavitation problems are the break-up regions of sheet cavitation and blade root cavitation, which may lead to serious erosion damage when the cavities implode close to the material surface. Also tip-vortex cavitation may lead to erosion on either the propeller blade or on the rudder, situated in the wake of the propeller. The chain of energy conversions that may ultimately lead to cavitation erosion is fascinating: the macroscopic cavities occurring as sheet and vortex cavitation contain potential energy as there exists a pressure difference between the contents of the cavity (vapour pressure) and the surrounding fluid. If either the spatial or the temporal pressure gradient (or both) become steep, the potential energy is converted into kinetic energy and when the kinetic energy of the surrounding fluid is sufficiently large, multiple shock waves occur in the final stage of the collapse of the cavity, where local pressures of up to some 10 000 times atmospheric pressure may occur at very local temperature rises up to some 800 K. At close inspection, the forms of cavitation that are involved in this energy cascade are almost always cavitating vortical structures.

Thrust breakdown, such as occurred during the first trials with the ‘Turbinia’ was caused by excessive sheet cavitation, limiting the suction side pressure of the blade to vapour pressure, thereby limiting the thrust production. This type of cavitation problem is well understood these days, and the most effective remedy is to distribute the propeller load over a larger surface, as was ultimately also applied by Parsons.

2.2. Phenomenological study of vortex cavitation

Tip-vortex cavitation can be viewed as a canonical problem that captures many of the essential physics associated with vortex cavitation in general [14]. In fact, much information can be obtained from water tunnel simulations of tip-vortex flow around stationary hydrofoils (figure 3). This presentation draws from experience gained from a wide range of experimentation with fixed geometry in numerous water tunnels scattered around the world with comparisons being drawn between cavitating vortices generated by foils and vortices from propellers.
The following will present a brief review of typical phenomena observed on cavitating vortices.

Figure 4 shows an example of a largely perturbed well-developed cavitating hub vortex, which was generated by a four-bladed swirl generator located just upstream of the measuring section of the high-speed cavitation tunnel at MARIN. The perturbations were observed as transient phenomena, apparently caused by small natural pressure disturbances in the tunnel flow.

Observations on developed cavitating tip and leading edge vortices are made in figures 5–7. Figure 5 shows the development of a cavitating vortex leaving the tip of a propeller blade, and being composed from a vortex sheet, rather than a single vortex (Figure 5b). The vortex sheet structure can be derived from the ‘twisted ribbon’, closely after the tip of the propeller blade, where the vortex sheet has not yet rolled up in a concentrated vortex. Further downstream (Figure 5a), we see that the vortex has much more developed into a cavitating vortex with a cylindrical cross section.
They compared sound data obtained in the same conditions and has been termed ‘vortex singing’ by Maines & Arndt [26]. They discovered that the sound data obtained in the same conditions could be identified as ‘vortex singing’. More recently, other laboratories have been able to detect the phenomenon in various cavitation conditions. The observation that singing only occurs over a very narrow range of cavitation number is difficult to reproduce without reasonable care. Only a slight change in the cavitation number is necessary to go from singing to no singing. This finding is in qualitative agreement with the observation that singing only occurs over a very narrow range of cavitation number.

The unstable behaviour of developed cavitating vortices is clearly observed in figure 8 where the developed cavitating tip vortex is seen to break up when the flow approaches the leading edge of the rudder horn.

2.3. Introduction to the radiated pressure signature from a propeller

It is well known that cavitating propellers cause radiated noise over a wide range of frequencies where the BPF forms a fundamental frequency of which multiples are often found back in the pressure spectrum. A typical example of hull pressure fluctuations measured on a plate field in the vicinity of the propeller is presented in figure 9. The blade passing frequencies can clearly be recognized from this pressure amplitude spectrum as the peaks, as well as the broadband part. This latter part is typically associated with cavitating vortices, but the exact mechanism has yet to be unveiled.

Figure 10 shows the phenomena that are associated with the time-wise development of vortex cavitation developing on a propeller blade passing through the wake peak behind a ship’s hull, where a strong variation in the blade loading occurs. The blade loading dynamics are the main cause for the cavity dynamics, which also translates into tip-vortex dynamics.

3. Experimental research on cavitating vortices

Developed cavitation in a trailing vortex has been studied in the past by Souders & Platzer [21], Arakeri et al. [22] and Arndt et al. [14]. The focus of these earlier studies was on the general flow features such as the vortex trajectory and the scaling of the vapour core radius with cavitation number. No mention is made of a discrete tone in early research except for Higuchi et al. [23], presumably because of the fact that the phenomenon takes place over a very narrow range of cavitation number and is difficult to reproduce without reasonable care. More recently other laboratories have been able to detect the same sound (e.g. [24, 25]).

This interesting anomaly is only observed under special conditions and has been termed ‘vortex singing’ by Maines & Arndt [26]. They compared sound data obtained in the same facility as used by Higuchi et al. [23] and in the larger water tunnel used by Arndt & Keller [27] in Obernach, Germany. When properly normalized, the results from the two facilities agree amazingly well. What is apparent is that when singing occurs it shows up in the spectrum as a very intense frequency peak of about 25 dB above the background noise. The phenomenon is very sensitive to cavitation number \( \sigma \) and water quality. Only a slight change in \( \sigma \) is necessary to go from singing to no singing.

A simple analysis highlights the essential features of the phenomenon. The experimental observations suggest a standing wave on the surface of the hollow vortex core (figure 13). Thomson [28] studied the wave pattern on a stationary, irrotational hollow core vortex. He found two dominant helical modes, one rotating with the same sense as the vortex and the other rotating and propagating in the opposite direction. As suggested by Keller & Escudier [29], a standing wave is possible when the vortex is superimposed on a uniform axial flow. This can occur when the celerity of the counter-rotating mode is equal and opposite to the free-stream velocity, \( U \). This occurs when \( f \lambda / U = 1 \), \( \lambda \) being the wavelength of the disturbance. Using these ideas, Maines & Arndt [26] found the surprising result that a standing wave will occur at only a single value of \( \sigma \). This finding is in qualitative agreement with the observation that singing only occurs over a very narrow range of cavitation number.

3.1. Early research

Initial studies were made in the USA and Germany in a collaborative programme of research that lasted several years. Four hydrofoils of elliptic planform with aspect ratio
3 but different cross sections were used for this study. The hydrofoil sections chosen were a NACA 662-415 \( a = 0.8 \), a modified NACA 4215 (designated herein as NACA 4215M), a NACA 16-020 and a NACA 66-012. Two sets of each foil were constructed. The smaller set had a root chord, \( c_0 \), of 81 mm and a half span, \( b \), of 95 mm while for the larger set \( c_0 = 129.4 \) mm and \( b = 152.4 \) mm. The small set was used for cavitation testing, force measurements and observation of the bubble dynamics at the St Anthony Falls Laboratory (SAFL). The larger set was used for cavitation studies and force measurements at the Versuchsanstalt für Wasserbau (VFW) in Obernach, Germany. The larger foils were also used for oil flow visualization studies at SAFL. These complementary studies provide a comprehensive view of the flow in the tip region and its correlation with cavitation.

Cavitation testing and force measurements were made in two water tunnels, one at SAFL which has a 190 mm square cross section \[14\] and the other at VFW with a 300 mm square cross section \[27\]. Thus span to tunnel size was held constant for the two series of experiments. Oil flow visualizations were obtained in two wind tunnels, one at the Department of Aerospace Engineering at the University of Minnesota \[30\] and the second at SAFL (originally an air model of the HYKAT facility, \[31,32\]).

Cavitation tests were performed by fixing the angle of attack and velocity and then slowly lowering the pressure until fully developed cavitation occurred, with the vapour-filled core just barely attached to the foil. Careful adjustments in either velocity or pressure were made until ‘singing’ occurred. It was found in initial trials that the singing phenomenon was far more reproducible with the NACA 66; series foil. Thus this study was concentrated on tests with this foil. Oil flow data were obtained using a spray of fine droplets of an oil and titanium-oxide mixture \[15\]. The wind tunnel was run at the test velocity of 56 m s\(^{-1}\) (Re \( \approx 485 \) 000). This technique highlighted the details of the boundary layer flow especially in the tip region.

Observations of cavitation were made with either conventional still photography or with high-speed video. Only still photography, using a standard Nikon 6006 camera, was used for observations at SAFL. A Kodak video camera with the possibility of framing rates as high as 40 500 fps was used at Obernach in conjunction with still photography, also using a standard Nikon camera. Video observations were made at a framing rate of 4500 fps, which was more than adequate for observing phenomena that had a frequency less than 600 Hz. Data were collected over a range of lift coefficients, velocities and water quality in both facilities.

Radiated sound was measured in the SAFL tunnel with a hydrophone positioned above the hydrofoil tip in a tank of quiescent water that was separated from the test section by a thin plate of Plexiglas \[23\]. The hydrofoil was mounted at the floor of the test section and the thin Plexiglas plate formed the roof of the test section. A similar set-up was used in Obernach, except that the hydrofoil was mounted in the roof of the test section and a single hydrophone was mounted in a tank of water that was positioned against one of the side windows of the test section. Therefore, the observation angle differed by approximately 90° in the two test facilities. The significant differences in the acoustic path for measurements in the two facilities precluded comparison of amplitude data. Only frequency data were compared. A serious attempt was

![Figure 10. Stills of high-speed video images of the collapse of the cavitating tip vortex where the non-uniform wake field in which the propeller operates is responsible for the cavity dynamics, Bosschers \[20\].](http://rsfs.royalsocietypublishing.org/)

(a) 54.5° blade position, (b) 71° blade position, (c) 85.7° blade position, and (d) 100.5° blade position.
made to acoustically calibrate the VFW tunnel. However, an accurate calibration procedure was not possible in the frequency range of interest, because of the complex acoustic response of the water tunnel.

3.2. Experimental observations

A sample comparison of the measured sound spectra with and without singing is shown in figure 11. These data were obtained in the Obernach tunnel by holding the velocity constant at 13 m s\(^{-1}\) and slowly lowering \(\sigma\) until singing occurred. The difference in \(\sigma\) for the two spectra is very small. What is apparent is that when singing occurs it shows up in the spectrum as a very intense peak of about 25 dB above the background at a discrete frequency.

The singing vortex has been observed primarily on the NACA 662-415 hydrofoil. Singing was also achieved with the larger scale NACA 4215M at the Obernach facility but only at very low amplitudes. Singing was not observed with the smaller scale NACA 4215M in the SAF facility. In general, the phenomenon has the appearance of a standing wave superimposed on the surface of the hollow vortex core. Figures 12 and 13 are photographs of a cavitating core without and with singing, respectively. In figure 12, the cavitation number has been raised just enough to suppress the singing. In figure 13, \(\sigma\) has been lowered to the point that the vortex begins to sing. Note the thickening of the core just downstream of the tip followed by a very thin core. This pattern is repeated downstream. As the oscillation progresses through one cycle, the radius of the thick portion near the tip decreases while the thin segment expands radially.

The driving mechanism for this phenomenon is still unknown; however, flow-induced vibration is discounted, in agreement with Higuchi et al. [23]. The natural frequency of both sets of foils was measured using a laser vibrometer. At the SAF facility, measurements were taken with the foil outside the tunnel. Thus, the measured frequency is slightly higher than it would be submerged in water. The natural frequency of the hydrofoil at the Obernach facility was measured while submerged in the water tunnel test section under static conditions. Hydrofoil vibration was then monitored with the laser vibrometer over a wide range of flow velocity with and without cavitation and with and without singing. Under all conditions, the predominant vibrational frequency was the same as that measured under static conditions. In both cases, the foil natural frequency was found to be much lower than the observed singing frequencies.

Throughout the test programme, singing was only observed when the hollow core was attached to the tip. Thus, a more likely mechanism for singing is the result of a complex interaction between the tip boundary layer and the attached cavity. High-speed video observations indicate that the attached tip cavity oscillates with the same frequency as the pulsating vortex. Figure 14 highlights the relationship between the

Figure 11. Comparison of noise spectra with and without singing, \(U = 13\) m s\(^{-1}\) (NACA 662-415 hydrofoil in the Obernach facility).

Figure 12. Photograph of developed tip-vortex cavitation without singing (NACA 662-415).

Figure 13. Photograph of singing vortex with enlarged core near the tip (NACA 662-415).
tip cavity and the average boundary layer characteristics. This picture was created by superimposing a single image of a tip cavity at a cavitation number slightly higher than \( \sigma_e \) with a photograph of oil film streaklines taken at an equivalent condition in a wind tunnel. Maines & Arndt [33].

The experimental observations suggest a standing wave on the surface of the hollow vortex core (figures 12 and 13). Thomson [28] studied the wave pattern on a stationary, irrotational hollow core vortex. He found two dominant helical modes, one rotating with the same sense as the vortex and the other rotating and propagating in the opposite direction. As suggested by Keller & Escudier [29], a standing wave is possible when the vortex is superimposed on a uniform axial flow. This can occur when the celerity of the counter-rotating mode is equal and opposite to the free-stream velocity, \( U_0 \). This occurs when \( \alpha v / U = 1 \), \( \lambda \) being the wavelength of the disturbance. Using these ideas, Maines & Arndt [26] computed the surprising result that a standing wave will occur at only a single value of \( \sigma \). This finding is in qualitative agreement with the observation that singing only occurs over a very narrow range of cavitation number. Despite the limitations of the theory, correlation of the frequency with measured vapour core radius was quite good. The most likely value is \( 2 \pi f_{ac} / U = 0.5 \) at \( \sigma_e \approx 1.2 \). However, imprecise lock-in can occur over a range of \( \sigma_e \) and \( \sigma_o \) given by \( 2 \pi f_{ac} / U = 0.45 \) \( \sigma_0 / 2 \) with \( 0.25 \leq 2 \pi f_{ac} / U \leq 0.65 \). This is shown in figure 15. The exact mechanism for singing is not understood. It appears that a standing wave on the vapour–liquid interface of the vortex core is in phase with sheet cavity oscillations when this phenomenon occurs.

3.3. Repeat experiment in Delft

The reference material from Maines & Arndt [26] provided a very good starting point to explain the origin of a cavity eigenfrequency. Owing to the state of the art at that time, the spatial resolution and contrast of the high-speed video recordings was insufficient to confirm the original theory by Thomson [28]. To better understand the part vortex cavitation plays in the generation of broadband pressure fluctuations the experiments of Maines & Arndt [26] with the NACA 66-415 \( a = 0.8 \) chord = 0.1256 m blade were repeated at the Delft University of Technology cavitation tunnel at \( \text{Re} = 0.9 \times 10^6 \), \( \sigma = 0.9–2.8 \) and \( C_i = 0.5–0.7 \) by Pennings et al. [34]. A sample result, shown in figure 16 [34], is comparable with figures 12 and 13 [26] except that the suction side is on the opposite side.

First, the time-averaged cavity size is given in figure 17. The open symbols are data taken from Maines & Arndt [26] at the SAFL facility. The closed symbols are taken at Delft by Pennings et al. [34]. The general trend of the cavity size \( r_c \) over the chord length \( C_i \) is higher for all cavitation numbers scaled with the lift coefficient. Owing to a limited free-stream velocity in Delft the free-stream pressure needs to be lowered to obtain the same cavitation number. As there is no special degassing facility the air content was rather high causing an increase in cavity radius due to diffusion of gas into the cavity. Some points at \( \alpha = 5 \) and \( \alpha = 7 \) are taken at the lowest air content possible. These points correspond to similar points in previous data from SAFL. In most cases a stationary wave was observed on the cavity. The wavelength of this shape is given in figure 18 in relation to the maximum cavity diameter.

In the case of vortex singing at the facility of Obernach, the frequency obtained from high-speed video recordings of the cavity correlated very well with the acoustic spectrum. For the recent results in Delft a comparison was made between the measured sound and the high-speed video recordings. The case described in detail is at \( C_i = 0.6, \ U_{ac} = 6.3, \ \sigma = 1.6, \ \text{Re}_c = 0.9 \times 10^6 \) and dissolved oxygen concentration 4 mg l\(^{-1}\). Although a large amplitude tonal frequency is observed on the cavity diameter no sound at the same frequency was observed. In the experiments in Delft no vortex singing was observed for any combination of lift coefficient, cavitation number and Reynolds number as mentioned in the work of Maines & Arndt [26]. It should be noted that the experiments at Delft by Pennings et al. [34] were made with a combination of the physical blade size of the Obernach tests with the Reynolds number of the SAFL test. This is expected to result in much lower cavity singing frequencies than the SAFL tests.

To still be able to better understand the underlying mechanisms the recent data from Delft by Pennings et al. [34] is related to the dispersion relation for waves on the vortex–cavity interface by Thomson [28] using small modifications as proposed by Boscchers [35].

An example of the three main cavity vibrational modes is presented in figure 19. The dispersion relation gives for one wavenumber two frequencies for each mode indicated in the
The variation of wavelength with core diameter as observed in three facilities, SAFL and Obernach from Maines and Arndt [26], Delft from Pennings et al. [34]. Reproduced from Pennings et al. [34].

The SAFL data correspond to singing Maines and Arndt [26], Delft data taken from Pennings et al. [26], and angle of attack 1

The SAFL data correspond to singing Maines and Arndt [26], Delft from Pennings et al. [34]. Reproduced from Pennings et al. [34].

The main features which stand out are marked. The straight dashed line is the simple convection of wave energy with amplitude is in a logarithmic scale with red being high values and blue the low values or the background signal. The main features which stand out are marked. The straight dashed line is the simple convection of wave energy with constant group speed 1.19 $U_{m}$. The zero frequency crossing corresponds to the stationary wave shape previously presented in figure 18. The features with equal slope are harmonics in the FFT analysis, which are faintly yellow.

Two other features can be identified which do not share this slope. The model for the cavity dynamics requires four parameters of the flow: the speed of sound in water $c$, the free-stream velocity $U_{m}$, the cavity radius $r_c$ and, finally, the angular velocity of the cavity edge. This last one is the only unknown in the current experiment. This parameter is used to match $n = 0^\circ$ mode and the $n = 2^\circ$ mode to the experimental data.

For a series of different experimental conditions and thus cavity radii $r_c$, a Lamb–Oseen velocity field model is matched to the found angular velocity at the cavity edge. The result is the viscous core radius $r_v$ in non-cavitating condition and the circulation of the tip vortex. The values found for both parameters are quite realistic as compared with data found by others with similar blades [36,37].

The cavity vibration model by Bosschers [35] is very well able to describe the three main deformation and displacement modes on the cavity surface. Using the simple viscous correction as proposed by Bosschers [35], the cavitation number can be related to the non-dimensional vortex singing frequency or cavity eigenfrequency.

The vortex singing frequency or the eigenfrequency of the cavity is hypothesized to be located at the point where the group velocity of the breathing $n = 0$ mode is zero. This corresponds to the location where the slope of the $n = 0$ mode is horizontal. See Pennings et al. [34] for detailed experimental results on the existence of clear dispersion relations for the three deformation modes and support for the cavity eigenfrequency at zero group velocity of the $n = 0$ mode.

Zero group velocity can be understood as the absence of convection of wave energy. Any wave energy at this
frequency will stay at the same location with respect to the blade tip. If energy would be supplied to this frequency the amplitude would grow strongly. At this negative frequency the phase velocity is negative which results in upstream travelling waves. These are expected to interact with the flow instability as seen in the boundary layer visualization of figure 14. As can be seen in figure 20 the line for convection of wave energy at 1.19 \( U_{\infty} \) crosses the dispersion relation line for the \( n = 0 \) mode at the location of zero group velocity. This could be a mechanism by which energy is transferred to the \( n = 0 \) mode, that under the right conditions might lead to amplification of diameter fluctuations.

Figure 21 is a comparison between the vortex singing cases in SAFL and Obernach, designated with open symbols, and the cases for which the cavity vibration model by Bosschers [35] has been verified in Delft by Pennings et al. [34], designated with filled symbols. The main difference is caused by the ratio between the cavity radius over the viscous core radius. In Delft this ratio is lower than in SAFL and Obernach. This is because of the significantly lower Reynolds number with respect to the Obernach experiments and a larger chord length with respect to the SAFL experiments both increasing the viscous core size. Thus, the viscous effect in the retardation of the rotational frequency is more significant in Delft resulting in lower frequencies for the same cavitation number.

The lines through the data points are based on estimates of the velocity field around the tip-vortex cavity. In the cases from Delft these follow directly from the obtained angular velocities of the experimental data. In case of the Obernach data, the same tip-vortex sheet roll-up parameter is used as in Delft for the same lift coefficient. The viscous core size is chosen such that the dashed line properly represents the vortex singing cases in Obernach.
4. Discussion

No singing vortex was observed in the experiments in Delft but all the properties of the tip-vortex cavity dynamics agree well with previous findings in SAFL and Obernach. The strongest current hypothesis for this difference is that a strong boundary layer flow or sheet cavity instability interacting with the tip-vortex cavity is necessary to amplify the eigenfrequency. The truncation of the trailing edge of the blade because of manufacturing limitations probably resulting in square edges gives a strongly defined separation point on the pressure and suction side of the blade. Because no instability could be present there is no amplification and no significant sound production.

The reason that the singing vortex phenomenon is very sensitive to flow conditions and is very hard to reproduce in different facilities has a strong connection to the relevance for full-scale ship propellers. All steady tip-vortex cavities have a distinct eigenfrequency. In case of steady uniform inflow this does not produce any audible sound, as is the case in Delft. Ship propellers are designed to mostly operate in the wake field of a hull. Therefore, excitation because of non-uniformity of the inflow is always present in some form. This very often results in excitation of the tip-vortex cavity and production of broadband pressure fluctuations. The part that is very reproducible in all the experiments performed in different facilities on the NACA 66°-415 wing with elliptical planform is the cavity eigenfrequency. The part that is very hard to reproduce which is almost always present on actual propellers in the form of inflow fluctuations is the excitation source. This last part determines if a singing vortex is observed in the model experiments and is irrelevant for full-scale propellers, because of the origin of excitation on model scale in the boundary layer or trailing edge flow. Thus, the singing vortex is an exceptional case that gives insight in the role of the vortex-cavity eigenfrequency in the centre frequency of broadband pressure fluctuations on full-scale ship propellers.

The cavity vibration model by Bosschers [35] was verified for the existence of the dispersion relation for the three main oscillation modes and the origin and value of a singing vortex or cavity eigenfrequency. The main model is based on potential flow and does not include viscous effects. As seen from the frequency-wavenumber diagram in figure 20 the descriptive strength of the dispersion relation of the various modes which are only based on the cavity angular velocity is quite good nonetheless.

The relation between the cavitation number and cavity radius based on the simple viscous correction assumes the cavity because of an oversaturation in the test conditions. For most of the data points in Delft compared with the SAFL results the expected cause of the increase in cavity radius is the diffusion of dissolved gas from the water into the cavity because of an oversaturation in the test conditions. There is room for improvement in correctly estimating the cavity radius based on the cavitation number and dissolved gas concentration or cavity pressure.

The final issue is related to the two-dimensionality of the model. The simplicity of the model is essential for a basic understanding of the underlying physics which is lost when more complicated CFD analyses are used. On average, the model is quite capable of describing a tip-vortex cavity in the process of roll-up using a stream-wise mean cavity angular velocity.

5. Conclusion and recommendations

Based on the experiments by Maines & Arndt [27] that were part of a larger research effort initiated by the additional optical analysis of the experiments in Delft, it is concluded that a cavitating vortex can be excited in its natural frequency under special circumstances. If this is done, it may lead to a strong tonal sound, a phenomenon called singing by Arndt et al. An important feature of the singing phenomenon is sensitivity to $\alpha$. In the early research by Maines & Arndt [26], it is suggested that a single frequency at a value of $2\pi f_{\alpha} / U = 0.5$ at $\alpha_r \approx 1.2$ is responsible for a singing vortex. However, imprecise lock-in can occur over a range of $\alpha_r$ and $\alpha_r$ given by $2\pi f_{\alpha} / U = 0.45 \alpha_r^{1/2}$ with $0.25 < 2\pi f_{\alpha} / U < 0.65$. More recent results by Pennings et al. [34] from the matching of an analytical model to the data of Delft indicate that the range of frequencies and cavitation numbers at which this happens can be explained by the condition where the group velocity of the monopole $n = 0$ mode equals zero. Here the negative phase speed results in upstream moving waves which interact with and are amplified by the connection of the cavity to the blade tip. If the vortex is not excited in its natural frequency, it does not produce a sound that could be detected for the given background noise in the cavitation tunnel.

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