Tiny bubbles challenge giant turbines: Three Gorges puzzle

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Since the birth of the first prototype of the modern reaction turbine, cavitation as conjectured by Euler in 1754 always presents as a challenge. Following his theory, the evolution of modern reaction (Francis and Kaplan) turbines has been completed by adding the final piece of the element ‘draft-tube’ that enables turbines to explore water energy at efficiencies of almost 100%. However, during the last two and a half centuries, with increasing unit capacity and specific speed, the problem of cavitation has been manifested and complicated by the draft-tube surges rather than being solved. Particularly, during the last 20 years, the fierce competition in the international market for extremely large turbines with compact design has encouraged the development of giant Francis turbines of 700–1000 MW. The first group (24 units) of such giant turbines of 700 MW each was installed in the Three Gorges project. Immediately after commission, a strange erosion phenomenon appeared on the guide vane of the machines that has puzzled professionals. From a multi-disciplinary analysis, this Three Gorges puzzle could reflect an unknown type of cavitation inception presumably triggered by turbulence production from the boundary-layer streak transitional process. It thus presents a fresh challenge not only to this old turbine industry, but also to the fundamental sciences.

1. Introduction

Following the invention of the reaction waterwheel by J.A. Segner in 1749 and the modern turbine theory developed by Euler [1], cavitation, as conjectured in [2], ‘is almost always an unwanted phenomenon in hydraulic machinery although it has some favourable effects in other fields. Actually, it is a main obstacle to the development of high-performance machines’ [3] (table 1).

To realize Euler’s prediction that this innovative reaction turbine could achieve almost 100% efficiency, a draft-tube must be installed underneath the runner to recover the remaining energy in the water leaving the runner. This final element has thus completed the configuration of the modern Francis and propeller turbines, significantly recovering the energy leaving from the runner, in particular for those high-specific-speed turbines with up to almost 42% of the total head recovered. However, along with other types of cavitation [7–9], the introduction of this draft-tube has caused pressure surges with accompanying cavitation in the draft-tube that becomes a major obstacle in turbine development, in particular for large Francis turbines under off-design operations. Extremely large amplitudes (up to 43.9%) of these oscillations at low frequencies down to 0.15 Hz [10] present real threats to the machine units, powerhouse and grid.

In the past two decades, in response to the demand for renewable and clean energies, hydro schemes of huge scale have been planned and built equipped with a new generation of giant Francis turbines ranging in capacity from 700 to 1000 MW. The world’s largest hydropower scheme, the Three Gorges project, is the first hydropower scheme having 24 units of 700 MW each installed in the left and right power plants. All these machines are designed and manufactured by the world’s leading manufacturers with cutting-edge technologies. They are the world’s largest Francis turbines in terms of their power (710 MW) and geometric dimensions (9800 mm of runner diameter). The 14 units in the left power plant manufactured by two consortiums (Alstom + HEC, GE + Voith
### Table 1. Notations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$D$</td>
<td>diameter [m]</td>
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<tr>
<td>$D_{in}$ and $D_{op}$</td>
<td>turbine-runner inlet diameters for model and prototype, respectively [m]</td>
</tr>
<tr>
<td>FPG and APG</td>
<td>favourite pressure gradient and adverse pressure gradient</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency [Hz]</td>
</tr>
<tr>
<td>$f_r$</td>
<td>rated rotation frequency of turbine [rotations per second]</td>
</tr>
<tr>
<td>GSB</td>
<td>group streak breakdown</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational constant $9.81$ [m s$^{-2}$]</td>
</tr>
<tr>
<td>$H$</td>
<td>water head of turbine [m]</td>
</tr>
<tr>
<td>$H_m$ and $H_p$</td>
<td>water heads for model and prototype, respectively [m]</td>
</tr>
<tr>
<td>$H_r$</td>
<td>suction head of turbine measuring from the runner exit to the tail-water surface [m]</td>
</tr>
<tr>
<td>$h_{hyd}$</td>
<td>hydraulic loss in the draft-tube [m]</td>
</tr>
<tr>
<td>$K_i$</td>
<td>cavitation number $K_i = (H_p - H_s - H')/(\rho V^2/2 g)$. Here, $H_s$ is the atmosphere pressure [m]; $H_p$ is the saturated vapour pressure [m]; and, $V_t$ is the area-averaged axial velocity at the inlet of the draft tube [m/s]</td>
</tr>
<tr>
<td>$k$</td>
<td>lumped elasticity of cavity in the draft-tube $k = dV/dH$ [m$^2$]. Here, $V$ is the cavity volume [m$^3$]; and $H$ is the pressure head [m]</td>
</tr>
<tr>
<td>$k^*$</td>
<td>dimensionless elasticity of cavity $k^* = K(H/D^2)$. Here, $k = k_{45} + k_{50}$ with $k_{45}$ and $k_{50}$ being passive and active elasticities, respectively; $H$ is the turbine head [m]; and, $D$ is the turbine runner diameter [m]</td>
</tr>
<tr>
<td>$L$</td>
<td>length [m]</td>
</tr>
<tr>
<td>$m$</td>
<td>dimensionless swirl number (rate) $m = \int_0^L r^2 v dv / L(\int_0^L r^2 v^2 dv)$. It reflects the strength of angular momentum flux to axial momentum flux. Instead of using the bulk parameter $Q$ as in the definition of $S$, integration of axial momentum flux is employed here [4]. This expression further considers the distribution effect of the axial velocity component on the axial momentum flux and is therefore often referred to in the studies of draft-tube flows</td>
</tr>
<tr>
<td>$m_{th}$</td>
<td>the theoretical value of the swirl rate $m$ calculated by the quasi-three-dimensional inviscid flow analysis [5]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>turbine unit capacity [KW or MW]</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>static pressure [N m$^{-2}$]</td>
</tr>
<tr>
<td>$p$</td>
<td>fluid pressure [N m$^{-2}$]</td>
</tr>
<tr>
<td>$\bar{p}(t)$</td>
<td>measured wall pressure pulsation in the draft-tube [N m$^{-2}$]; for a shallow draft-tube, it is consists of three components. That is, the component induced by the vortex-rope precession $\bar{p}_f(f_o - t)$, the component induced by the oscillation of recovery pressure at the foot of the bend $\bar{p}_r(f_o - t)$ and the system pulsation component caused by cavitation $\bar{p}_c(f_o - t)$. For a deep draft-tube, it consists of a different set of three components. That is, the component induced by the vortex-rope precession $\bar{p}_f(f_o - t)$, the component induced by the oscillation of recovery pressure at the foot of the bend $\bar{p}_r(f_o - t)$ and the system pulsation component caused by cavitation $\bar{p}_c(f_o - t)$</td>
</tr>
<tr>
<td>$Q$</td>
<td>flow rate [m$^3$ s$^{-1}$]</td>
</tr>
<tr>
<td>$Re_L$</td>
<td>boundary-layer-based Reynolds number $Re_L = U L/\nu$. Here, $U$ is the free-stream velocity; $L$ is the distance in the x direction; and $\nu$ is the kinematic viscosity [m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$Re_{Lcri}$</td>
<td>critical Reynolds number for boundary-layer flow transition $Re_{Lcri} = U L_{cri}/\nu$. Here $U$ is the free-stream velocity at the transition [m s$^{-1}$]; and, $L_{cri}$ is the distance in the x-direction where boundary-layer transition occurs [m]</td>
</tr>
<tr>
<td>$S$</td>
<td>defined by Benjamin [6] as flow force $S = \rho U^2 h + (1/2)gh^2$ [kg s$^{-2}$]. Here, $U$ is the horizontal velocity [m s$^{-1}$]; and $h$ is the water depth [m]</td>
</tr>
<tr>
<td>$S_t$</td>
<td>designated as a dimensionless swirl number (rate) $S_t = 120/\rho U^2$, referring to equation (3.3). It reflects the strength of the angular momentum flux to the axial momentum flux</td>
</tr>
<tr>
<td>$S_i$</td>
<td>dimensionless frequency of pressure surges, i.e. Strouhal number $S_i = 2R_i/\omega_i$. Here, $R$, $f$ and $\omega$ are the radius, the frequency and the averaged velocity, respectively, evaluated/calculated for the inlet of the draft tube as indicated by its subscript of $i$</td>
</tr>
<tr>
<td>$T_b$</td>
<td>period of bubble oscillation [s]. Then $1/T_b$ is the frequency of bubble oscillation [Hz]</td>
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(Continued.)
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<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$u_1$</td>
<td>translational velocity at the runner entrance $u_1 = \pi D_1 \omega$ [m s$^{-1}$]. Here, $D_1$ is the diameter of the turbine runner inlet [m]; and $\omega$ is the angular velocity of turbine runner rotation [rad s$^{-1}$]</td>
</tr>
<tr>
<td>$u_2$</td>
<td>translational velocity at the runner exit $u_2 = \pi D_2 \omega$ [m$^{-1}$ s]. Here, $D_2$ is the diameter of the turbine runner outlet [m]; and $\omega$ is the angular velocity of turbine runner rotation [rad s$^{-1}$]</td>
</tr>
<tr>
<td>$u'$</td>
<td>velocity perturbation in the cylindrical coordinate $u' = u(r) \exp[(\alpha r + m \theta + k z)]$. Here, $\alpha$ is complex frequency; $m$ and $k$ are the wavenumbers in the angular and axial directions, respectively</td>
</tr>
<tr>
<td>$V$</td>
<td>control volume [m$^3$]</td>
</tr>
<tr>
<td>$V_{tr}$, $W(r)$</td>
<td>azimuthal and axial velocities, respectively, in swirl flow [m s$^{-1}$]. $V(r) = (K/r) \left[ 1 - \exp(-\alpha r^2) \right]$ and $W(r) = W_1 + W_2 \exp(-\alpha r^2)$. Here, $K$, $W_1$, and $W_2$ are coefficients; and $\alpha$ is the common parameter</td>
</tr>
<tr>
<td>$V_{a1}$</td>
<td>azimuthal component of water velocity at the runner entrance [m s$^{-1}$]</td>
</tr>
<tr>
<td>$V_{a2}$</td>
<td>azimuthal component of water velocity at the runner exit [m s$^{-1}$]</td>
</tr>
<tr>
<td>$V_r$</td>
<td>flow velocity at the turbine runner exit [m s$^{-1}$]</td>
</tr>
<tr>
<td>$V_s$</td>
<td>water velocity at the exit of the draft-tube [m s$^{-1}$]</td>
</tr>
<tr>
<td>$v'$</td>
<td>velocity perturbation [m s$^{-1}$]</td>
</tr>
<tr>
<td>$y^*$</td>
<td>a non-dimensional wall distance for a wall-bounded flow defined as $y^* = u_<em>/v$. Here $u_</em>$ is the friction velocity at the nearest wall $u_* = \sqrt{\tau_w / \rho}$ [m s$^{-1}$]; $\tau_w$ is wall shear stress $\tau_w = \mu(\partial u/\partial y)_{y=0}$ [kg m$^{-1}$ s$^{-2}$]; and $y$ is the distance to the nearest wall [m]; and $\mu$ is the local dynamic viscosity [kg s$^{-1}$ m$^{-1}$]</td>
</tr>
<tr>
<td>$\Gamma_1$</td>
<td>circulation of water at the runner entrance $\Gamma_1 = \pi D_1 V_{a1}$ [m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>circulation of water at the runner exit $\Gamma_2 = \pi D_2 V_{a2}$ [m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>divergence of the tube [degree]</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>boundary-layer displacement thickness $\delta^* = 1.7208 \ell \sqrt{R_i}$ [m]. Here, $L$ is the length of the boundary layer; and $R_i$ is the boundary-layer-based Reynolds number</td>
</tr>
<tr>
<td>$\eta$</td>
<td>turbine efficiency that consists of hydraulic, volumetric and mechanical efficiencies: $\eta_h$, $\eta_v$, and $\eta_m$</td>
</tr>
<tr>
<td>$\lambda^*$</td>
<td>span-wise wavelength of the boundary-layer streaks $\lambda^* = 10 \delta^<em>$ [m] with $\delta^</em>$ being the displacement thickness [m]</td>
</tr>
<tr>
<td>$\lambda_{GB}$</td>
<td>averaged span-wise wavelength of GSB spots [m]</td>
</tr>
<tr>
<td>$\lambda_{strip}$</td>
<td>averaged span-wise wavelength of erosion strips [m]</td>
</tr>
<tr>
<td>$\lambda_{break}$</td>
<td>averaged span-wise wavelength of cavitation streaks [m]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity $\mu = \nu \rho$ [kg s$^{-1}$ m$^{-1}$]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity $\nu = \mu / \rho$ [m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>water density [kg m$^{-1}$]</td>
</tr>
<tr>
<td>$\Delta \psi_{RMS}$</td>
<td>the dimensionless draft-tube surge $\Delta \psi_{RMS} = \Delta H / (\Omega^2 / 2 g)$. Here, $\Delta H$ is the pressure surge in the draft-tube [m]; and $u_1$ is the area-averaged inlet velocity [m s$^{-1}$]</td>
</tr>
<tr>
<td>$\Delta \psi_{rot}$</td>
<td>the dimensionless rotational component of draft-tube surge $\Delta \psi_{RMS}$</td>
</tr>
<tr>
<td>$\Delta \psi_{syn}$</td>
<td>the dimensionless synchronous component of draft-tube surge $\Delta \psi_{RMS}$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>flux of angular momentum into the tube $\Omega = \int \rho \omega r w , ds$ [kg m$^2$ s$^{-2}$], which is often used as a reference quantity for swirl flows. Here, $S_1$ is the cross section of entrance; $w$ and $v$ are the velocity components in the axial and azimuthal directions, respectively</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity of turbine [rad s$^{-1}$]</td>
</tr>
<tr>
<td>$n_s$</td>
<td>turbine specific speed $n_s = n \sqrt{H} / \left[ \Omega^{3/2} / m^{3/4} \right]$. Here, $n$ is the rotation speed [r.p.m.]; $\Omega$ is the output [kw]; and $H$ is the water head [m]</td>
</tr>
<tr>
<td>$n_s'$</td>
<td>the unit speed $n_s' = nD / \sqrt{H}$ [r.p.m. m$^{-1/2}$]. Here, $D$ is the runner inlet diameter often designated as $D_1$ [m]. Though it is a dimensional parameter, the unit-speed equality is equivalent to the Strouhal number equality</td>
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</table>

In current practice within the turbine industry, two turbine flows are considered to be dynamically similar if their specific speed $n_s$, unit speed $n_s'$, unit flow rate $Q'$, and cavitation number $K$ are identical. These parameters are defined as follows:

- $n_s' = nD / \sqrt{H}$ [r.p.m. m$^{-1/2}$]. Here, $D$ is the runner inlet diameter often designated as $D_1$ [m]. Though it is a dimensional parameter, the unit-speed equality is equivalent to the Strouhal number equality
Siemens + DFEM) were all commissioned during the period from June 2003 to September 2005. However, immediately after commissioning, a very unusual and strange pattern of erosion never previously reported or studied has developed on the guide vanes made of stainless steel, as shown in figure 1. The erosion occurs in the form of flow-directional strips covered by a corroded rough surface. There are also signs of heating effect. These unusual patterns and strange appearances are entirely different from any type of known erosion patterns, including cavitation erosion. Questions and confusion, such as why the stainless steel becomes corroded and how steel could be heated in water, arose during and after a specially called in situ meeting (held on 12–14 March 2006) [12]. Various possible types of erosions were proposed such as corrosion, galvanic erosion, silt erosion, material deficiencies and manufacturing problems, etc., but not cavitation. On 19 March 2006, the author inspected the no. 11 (11F) unit, in particular the erosion on the representative no. 4 guide vane, and proposed a multi-disciplinary process that could be the underlying mechanism [11]. This analysis points to a possibly new cavitation inception mechanism that could be triggered by the boundary-layer streak transition while the boundary-layer receptivity is much higher to the low-frequency perturbations in free stream. These low-frequency pressure fluctuations could be mainly induced by a wrongly designed guide plate and further intensified through the manifestation and modulation process of draft-tube surges.

Having introduced the Three Gorges puzzle and the proposed hypothesis, this paper presents investigations of the validity of the hypothesis with a view towards exploring the possible root of the Three Gorges puzzle. The hypothesis initially proposed in 2006 is thus further developed with new concepts and evaluated by revisiting an unexplainable observation on foil cavitation at Caltech (USA). However, our fight against cavitation is far from over yet, as we still lack a good understanding of these tiny bubble nuclei when they interact with the complex flow structures encountered in the boundary layer of these giant turbines. This article concludes with the evaluation of the hypothesis validity as well as the significance of this puzzle.

2. Three Gorges puzzle

The strange erosion on the Three Gorges turbines was first spotted on the 11F turbine incidentally on 14 October 2005 (operated for 10 243.78 h) while examining and repairing the damaged guide plate. Later on, similar erosions were found on other units. All these 14 machines in the left plant developed more or less similar erosions. The machine has a maximum efficiency of 96.26%, rated speed 75 r.p.m. and head 80.6 m (min. head 61.0 m and max. head 113.0 m). At most times, the 11F unit had been operating under low head conditions (around and less than 70 m).

2.1. Appearance

The erosions occur only on the foil’s lower surface in the form of horizontal strips, starting from the favourite pressure gradient (FPG) zone and extending into the adverse pressure gradient (APG) zone, as shown by figures 1–3. The depth of the most eroded area is much less than 1 mm, which is covered fully by a corroded rough surface. Heated spots (figure 4), heated strips and heated tails of eroded strips are observed, which is another common feature [11]. These erosion strips are approximately in the direction of flow there. Another feature is the wedged head that is almost always present. A typical strip is shown in figure 3. The erosion strips are distributed span-wise showing regularities (figure 1), with an average span-wise spacing of approximately 0.100 m. The material used for 11F guide vanes is X5CrNiMo13-4 (EN-1088), which is close or equivalent to CA6MN (12.9% Cr, 4% Ni and 0.04% C), a commonly used martensitic stainless steel for turbine fabrication [13].

The types of damage proposed during the in situ meeting [12] were focused on various corrosions because of the obvious corrosive features. However, this resulted in the following questions: how can a stainless steel be corroded and why has corrosion occurred only on a selected area, not on the entire surface contacting with water? Material defects resulting from the manufacturing process (rolling of the steel sheet) were also proposed, but they could not explain why corrosion only occurred on the lower surface of the
foil and not on the upper surface. Silt erosion was a favourite proposal raised by the manufacturers, but the damage pattern suggests differently. Galvanic corrosion was also raised but without evidence. Almost every type of damage but cavitation was considered, because its appearance was obviously different from known patterns of cavitation erosion and the average static pressure there was higher than vapour pressure. This inconclusive meeting ended with such a ‘Three Gorges puzzle’.

2.2. Hypothesis

Based on the analysis depicted in the following sections, the initial conjecture [11] could be further developed into a reasonable hypothesis. That is, cavitation inception could be triggered by boundary-layer transition and associated turbulence production. The formation of boundary-layer streaks from the Tollmien–Schlichting (T–S) wave and its subsequent evolution and breakdown all involve significantly vortical dynamics generating extremely large velocity and pressure fluctuations at much lower frequencies than those of nuclei. The high perturbations of low frequencies in the free stream originated from the guide plate and significantly manifested by the severe draft-tube surges in giant turbines have thus further promoted the above process, making the tiny bubble nuclei trapped in the boundary layer much more prone to cavitation, in particular at the spots where group-streak breakdown (GSB) occurs. Once the first eroded spot is created, a dynamic process follows: the first spot could serve well as a spot of surface roughness, triggering subsequent (and/or enhancing the existing) cavitation and causing erosion immediately downstream. This dynamic and sustainable process progresses stream-wise, resulting in such a horizontal and equal-width erosion strip with a wedged head. Their span-wise distribution thus reflects the span-wise stochastic characteristics of the boundary-layer transitional process.

3. Severely low-frequency pressure surges

In the past two decades, turbine design has been undergoing a significant evolution that is capable of converting water energy to mechanical energy at an efficiency of almost 100% by employing as high a specific speed and as large a unit capacity as possible for given site conditions. As a result, giant fast Francis machines of extremely compact design have been developed. Consequently, severe pressure surges and instabilities at low frequencies are thus further escalated that not only pose risks to safe operation but also induce unknown phenomena and problems. The ‘Three Gorges puzzle’ is just such a consequence that is promoted by these low-frequency surges. To understand the severely low-frequency pressure surges encountered by the Three Gorges turbines, the surrounding context such as Euler’s theories, vortex breakdown in swirl flows and draft-tube surges will be briefly overviewed first.

3.1. Euler’s theories

In the eighteenth century, the 2000-year-old design of waterwheels experienced a rapid evolution, completing the whole process from the undershot impulse and overshot gravity wheels to the pioneering prototype of the modern reaction turbine. During this process, Euler laid down the foundation of the modern turbine theory and conjectured the possible cavitation phenomenon in water turbomachines. This theory has established that the theoretical efficiency for the reaction wheel could approach 100% under ideal conditions, i.e. a full recovery of the energy retained in the water leaving the waterwheel [15].
The fundamental equations for turbomachinery named after Euler determine the head \( H \) created by the pump or used by the turbine based on the conservation of angular momentum. For turbines

\[
H \eta_G = \frac{\omega I_1}{2\pi} - \frac{\omega I_2}{2\pi} = \frac{\omega}{2\pi}(I_1 - I_2).
\]

This determines the inlet- and outlet-circulation values of \( I_1 \) and \( I_2 \) for a given operating condition \( (H, \eta, \omega) \). Note that the recovered energy from the water leaving at the runner exit by installing the draft-tube is not explicitly reflected in the equations. Physically, the wasted head and the kinetic energy retained in the water leaving the exit are both recovered by the draft-tube through creating a vacuum pressure at the runner exit that increases the turbine velocity coefficient. The draft-tube recovers the suction head \( H_s \) and the kinetic head \( (V_2^2 - V_3^2)/(2g) \) at the cost of a usually smaller hydraulic loss \( h_{dt} \).

\[
H_{rec} = H_s + \frac{V_2^2 - V_3^2}{2g} - h_{dt}.
\]
Thus, installing the ‘draft-tube’ has created a new problem, i.e. the draft-tube surges and accompanying cavitation, in particular for Francis turbines. One of the advantages of hydraulic turbines in electricity supply grids is their capability of responding quickly to the variable demands from consumers connected to the grids. This requires turbines to operate in a wide range of loads from very low to over full load for those generating units undertaking a control role in the electricity grid. This poses a particular challenge to the Francis turbine because of its fixed blades that cannot control the worsening exit flow from the runner during off-design operations. This flow entering the draft-tube will further intensify the very complex hydraulic phenomenon there that is a highly unsteady flow full of large-scale vortices, strong shear and turbulence production, stagnations, reverse flows and vortex rope, etc. These flow structures could be further significantly complicated by cavitation. Accompanying these flow structures are pressure fluctuations of low frequencies and high amplitudes that cause structural vibrations and even grid power surges.

3.2. Vortex breakdown and draft-tube surges
Draft-tube surges contribute to the free-stream pressure perturbations, in particular the low-frequency components in the whole flow system of Francis turbines for the full operation range. Rheingans [16] laid down the basis for understanding the origin of these surges. He showed that the vacuum surge in the draft-tube with a frequency relating to the runner rotating speed is \( S = \text{r.p.m.}/3.6 \), reflecting the manifested swirling flow under part load conditions. The vortex breakdown in swirl flows is the first step towards understanding draft-tube surges.

3.2.1. Vortex breakdown in swirl flows
The significant structure change caused by vortex breakdown usually occurs in most flows dominated by longitudinal vortices. The case ‘vortex breakdown’ is mostly pertinent to the flows in the draft tube. Although earlier studies were done by [17–20], it was the experimental studies in both air and water [21] that gave a unified descriptive theory for the breakdown as being attributed to the swirling flow.

A gross parameter descriptive of the relative rotational and axial characteristics of the flow has been developed for correlating flow status [22]:

\[
\frac{\rho Q^2}{D} = \frac{f D^3}{Q} \frac{d}{d\vartheta} \int_{\gamma} \tilde{r} \tilde{v} \tilde{d} \vartheta - 1
\]

\[
+ \left( \frac{\rho Q^2}{D} \right) \left\{ \int_{\gamma} \tilde{r} \tilde{w} \tilde{v} \tilde{d} \vartheta + \int_{\gamma} \tilde{u} \tilde{v} \tilde{d} \vartheta \right\}
\]

\[
= - \left( \frac{P_0 D^3}{\Omega} \right) \int_{\gamma} \frac{\tilde{r} \tilde{v} \tilde{d} \vartheta}{\beta} d\vartheta.
\]

(3.3)

There are four dimensionless groups: the swirl number \( \Omega D / \rho Q^2 \), the Strouhal number \( f D^3 / Q \), the Euler number (dimensionless surge amplitude) \( P_0 D^3 / \Omega \) plus a relative tube length \( L / D \) (to be more general). Their experimental data of frequency and pressure are well correlated through dimensionless groups. As an example, the frequency tends to be constant for a particular value of \( \Omega D / \rho Q^2 \). Both dimensionless frequencies and pressures increase rapidly with the increase of \( \Omega D / \rho Q^2 \). Collated data [22] from vortex breakdown experiments of [19,23] show that the critical values of \( \Omega D / \rho Q^2 \) increase linearly with relative length \( L / D \), indicating the giant turbines (i.e. smaller value of \( L / D \)) to be more prone to vortex breakdown at given swirl numbers.

The effects of a diverging tube and adverse pressure gradient, which is pertinent to the case of a draft-tube cone, on vortex breakdown have been investigated in [24–26]. Three types of vortex breakdown are classified with clear flow structure demonstrated in a diverging cylindrical tube: (i) mild (double helix) breakdown, (ii) spiral breakdown followed by turbulent mixing, and (iii) axisymmetric breakdown followed by a thicker vortex core, then a spiral breakdown and finally turbulent mixing. The type and location of these stationary breakdowns are dependent on the Reynolds and circulation numbers. Figure 5 shows the evolution from the inception of spiral vortex breakdown towards axisymmetric vortex breakdown. In the hysteresis region, the filament does not spread out suddenly and symmetrically in the form of a ‘tulip’ but rather maintains its integrity for a short distance on one side of the reversed flow and then expands into an undular tape (figure 6). This hysteresis region exists for every constant circulation line in which there are two stable breakdown conditions as marked by the dotted lines in figure 7. This hysteresis phenomenon has also been discovered in turbines [27]. For larger swirls, breakdown moves progressively upstream with a bubble becoming a smooth and symmetric body.

For the internal structures of the vortex breakdown, following earlier studies [28,29], detailed measurements [30–32] reveal much of the physics of the phenomenon. For the approach flow, the axial velocity distribution has a jet-like shape that manifests with the increase of vane angle, whereas the swirl velocity shows typical features of a viscous core surrounded by irrotational flows, supporting the assumptions employed in most studies. That is, the azimuthal velocity \( V \) fits a Burgers vortex \( V(r) = (K/\rho)[1 - \exp(-a r^2)] \), whereas the axial velocity is fitted well by the function of \( W(r) = W_1 + W_2\exp(-a r^2) \) with the common parameter \( a \). The axial velocity decreases very slowly up to the stagnation point, where its value at the central line drops rapidly. The complexity of the flow details in the recirculation zone where low-frequency motions are dominant has been clearly revealed (figures 6b and 8). The axial and azimuthal velocities vary significantly in both space and time, forming four stagnation points along the axis (figure 8) and multi-cells in the axisymmetric bubble (figure 9). However, these velocities are much smaller than the approach flow. The dominant frequency of oscillation was as low as 2 Hz.

However, divergence does not always move the breakdown further upstream, in particular if the divergence is beyond a limit where the boundary-layer development may give rise to reverse flow on the tube wall. The experimental investigation [26] (figure 10) shows the influences of divergence, circulation and Reynolds number on the breakdown position. Note the overwhelming effect of boundary reverse flow for a divergence of \( y \) more than 2.36, which starts moving the breakdown position downstream instead of upstream. For turbines, this is often the case where the divergent angle is much larger (in the range of 12–13°) for maximizing energy recovery. The consequences of the divergence effect for the breakdown position for turbines is not clear yet.

With regard to the mechanism of vortex breakdown, two different mechanisms are suggested: the hydrodynamic
instability and the finite transition to a conjugate flow state as demonstrated, respectively, by the small perturbation analysis [33] and the ‘trapped wave’ theory [34].

The theoretical work [6] on the criticality obviously supports the mechanism of transition (to conjugate flow state). Benjamin derived a set of properties analogous to those of hydraulic jump by introducing a new quantity $S$ named as ‘flow force’, which is the sum of the horizontal momentum flux and the pressure force per unit span,

$$S = \rho (u^2h + (1/2)gh^2).$$

For any pair of conjugate flows, $S$ is greater for a subcritical than for a supercritical state. This is referred to as ‘flow force deficiency’ of supercritical flow. Given a stationary axisymmetric perturbation $e\phi(y)$ to this frictionless axisymmetric swirling flow $\Psi(y)$, the total stream function $\psi(x,y)$ then takes the form

$$\psi(x, y) = \Psi(y) + e\phi(y)e^{iy}.\tag{3.4}$$

Here, if $\gamma$ is a pure imaginary $ia$, this indicates a standing wave of wavelength $2\pi/a$. Benjamin derived that $\phi(y)$ must be a solution of a second-order ordinary differential equation (ODE), the Sturm–Liouville system

$$\phi_{yy} + \left( \frac{\gamma^2}{2y} - \frac{W_{yy}}{W} + \frac{I_y}{2y^2W^2} \right) \phi = 0,$$  

with boundary conditions $\phi(0) = 0$ and $\phi(a) = 0$. As the factor multiplying $\gamma^2$ in the ODE above is positive, there are infinite real eigenvalues $\gamma_0^2, \gamma_1^2, \gamma_2^2, \ldots$ allowing for solutions $\phi_0, \phi_1, \phi_2, \ldots$. But there is a limited possibility of standing waves, that is, with $\gamma^2 < 0$ such that $\gamma = ia$. Therefore, Benjamin argued that flow is supercritical if, for all eigenvalues, $\gamma^2 > 0$ and subcritical if, for at least one eigenvalue, $\gamma^2 < 0$.

For the integration of flow force being stationary for weak variations with fixed end-points, $\psi$ should satisfy the Eulerian characteristic equation

$$\frac{d}{dy} \left( \frac{\partial f}{\partial \psi} \right) - \frac{\partial f}{\partial \psi} = 0.$$

Benjamin demonstrated conjugate solutions in the plane $(\psi, y)$ with the supercritical solution giving a minimum of $S$.

Figure 5. For a given Reynolds number (5000) with swirl circulation (vane angle measured) increasing slightly from 30°, the evolution of spiral vortex breakdown to axisymmetric vortex breakdown in three stages: (a) initiation of spiral breakdown, (b) spiral breakdown prior to inception of the bubble, and (c) the nascent state of axisymmetric vortex breakdown [24].
and the conjugate solution giving neither maximum nor minimum. However, a much simpler rule that does not need detailed information, such as the axial and azimuthal velocity distributions, has been proposed [35]: for a swirl flow to be supercritical, \( v/w < 1 \) must exist at the location of maximum swirl.

### 3.2.2. Draft-tube surges

It was Cassidy & Falvey [22] who for the first time proposed the manifestation of vortex breakdown as being the underlying mechanism for draft-tube surges. However, the phenomenon of draft-tube surges is far more complex. First, the geometry is more complex than a divergent tube owing to the inclusion of a 90\(^\circ\) bend followed by another horizontal diffuser. The highly non-axisymmetric condition imposed by the bend and encountered by the flow passing through there will trigger new modes of oscillations and enhance existing ones. For the cone section, the diffusive angle is designed much larger. Apart from promoting flow separation, more energy is obtained from the flow deceleration (equation (3.2)) to feed the trapped wave, promoting violent surges. For the Three Gorges case, the entering flow is further complicated by the "guide plate", which will be discussed later. Third, a cavitated vortex core is a common scenario in Francis turbines, and the cavitating vortex breakdown is a much less understood phenomenon owing to its complicated nature.

Earlier works [36–38] suggest the spiral vortex as being a flow structure causing the self-excited oscillations in the draft-tube. For cavitated cases, the entrained vapour and vapour/air mixture change the surge behaviour, deviating from the single-phase swirl flows featured by increased elasticity (or compliance) as first recognized in [39].

For a bending draft-tube, the flow becomes extremely complex. The elbow interacts with the processing inhomogeneous pressure field associated with the spiral vortex core and triggers the periodical formation of a large separation/reverse flow zone on the top of the elbow immediately after the corner turn. This periodic change of energy-recovering efficiency, which is caused by the change of hydraulic loss \( h_{\text{fl}} \) (equation (3.2)), involves a significant portion of energy explored by the high-specific-speed Francis turbines and oscillates at a low frequency. Therefore, it serves well as an exciting resource for surges to occur globally throughout the whole system. This phenomenon was first
recognized in [4]. Nishi et al.’s pioneering experiments in parallel with the numerical study performed by Doefler [40] are reflected in [4,5,41]. Nishi’s experiments clarify the surge mechanism. That is, apart from the rotational surge, the synchronous and severe pressure surge at half-load for Francis turbines with an elbow-type draft-tube recognized as resonance is generated through the interaction between the vapour/gas volume in the cavitated vortex core and the oscillation of the pressure recovery caused by the precession of the spiral vortex core near the entrance of the elbow (figure 11). This shares and experimentally proves Doefler’s viewpoint of the passive and active compliances assigned to the symmetric vortex core upstream and the spiral-shaped vortex downstream, respectively, in [40] (figure 12). The flow status in the model elbow-type draft-tube is measured by two parameters: the swirl rate \( m \) and the cavitation number \( K_i \). The theoretical value \( m_{th} \) of the swirl rate \( m \) is determined by the quasi-three-dimensional inviscid flow analysis [5]. The dimensionless frequency of pressure surges, i.e. the Strouhal number, is

\[
S_t = \frac{2Rf}{u_{in}}.
\]  

(3.7)

Representative samples of temporal surges for flow regime III, i.e. a corkscrew-type vortex core in the draft-tube, have been analysed. The origination of the vortex rope was proposed as illustrated in [5]. The data of \( L_1 \) and \( L_2 \) measuring points (in 90° difference) with and without cavitations \( (K_i = 8.5 \text{ and } 25.0, \text{ respectively}) \) have shown a 90° phase shift (for non-cavitation) indicating a rotationary pressure surge (named as QR type) and virtually no phase shift (for cavitation) indicating a synchronous pressure surge (named as QS type). For both cases, the essential frequency upstream of the bend is equal to the precession frequency of the spiral vortex core. For the shallow draft-tube, the measured wall pulsation \( \tilde{p}(t) \) can be decomposed as

\[
\tilde{p}(t) = \tilde{p}_{ro}(f_{ro} \cdot t) + \tilde{p}_{sy}(f_{ro} \cdot t) + \tilde{p}_{si}(f_i \cdot t).
\]  

(3.8)

Here, \( \tilde{p}_{ro}(f_{ro} \cdot t) \) is the pulsation component induced by the precession of the vortex rope in the measuring section (i.e. QR type) where \( f_{ro} = f_{ro-i} = f_{ro-b} \), \( \tilde{p}_{sy}(f_{ro} \cdot t) \) is the pulsation component induced by the oscillation of recovery pressure at the foot of the bend that has the synchronous attribute (i.e. QS type), and \( \tilde{p}_{si}(f_i \cdot t) \) is the system (i.e. the whole draft-tube for the draft-tube component tested here) pulsation component caused by cavitation that has a frequency corresponding to the natural frequency \( f_s \) of the system. For the deep draft-tube

\[
\tilde{p}(t) = \tilde{p}_{ro}(f_{ro-i} \cdot t) + \tilde{p}_{ro}(f_{ro-b} \cdot t) + \tilde{p}_{si}(f_i \cdot t).
\]  

(3.9)

Here, \( \tilde{p}_{ro}(f_{ro-i} \cdot t) \) is the pulsation component induced by the precession of the vortex rope in the measuring section (i.e. QR type), but \( f_{ro} = f_{ro-i} = f_{ro-b} \) no longer exists; \( \tilde{p}_{ro}(f_{ro-b} \cdot t) \) is the pulsation component induced by the oscillation of recovery pressure at the foot of the bend (i.e. QS type), and \( \tilde{p}_{si}(f_i \cdot t) \) is the system pulsation component caused by cavitation.

A typical relationship of surge with cavitation is shown for a shallow draft-tube in figure 13. Cavitation appears around \( K_i = 15 \) and starts amplifying the surge \( \Delta \phi_{RMS} \), leading to a peak around \( K_i = 8.5 \) (i.e. the critical cavitation number). By separating pulsations measured from \( L_1 \) and \( L_2 \) sensors as the rotational component \( \tilde{p}_{ro} \) and synchronous component \( \tilde{p}_{sy} \) [42,43] (figure 14), the rotational component \( \Delta \phi_{ro} \) virtually remains independent of cavitation number \( K_i \) whereas the synchronous component \( \Delta \phi_{sy} \) is amplified (once cavitation appears) to a peak at the critical cavitation number \( K_i = 8.5 \) when both frequencies coincide (i.e. \( f_i = f_{ro-b} \) and system resonance occurs. This indicates that the surge increase from cavitation is mainly attributed to the synchronous component with a resonance feature. The observation [41] shows that, during the synchronous surge, the volumetric size of the cavitated vortex core varies periodically with circumferential positions. At the positions of the low inlet pressure or the low bend pressure, the core enlarges, whereas at the positions of the high inlet pressure or the high bend pressure the core shrinks (figure 11). For a deep draft-tube, owing to the much longer inlet cone, the swirl strength decays more towards the downstream, which causes rope precession frequency decreases more at the elbow entrance [44]. He also did a typical spectral analysis showing three distinguishing frequencies in deep draft-tube flow and their dependence on cavitation number \( K_i \). All their features comply with the above decomposition of the measured wall pulsation \( \tilde{p}(t) \). In particular, the frequency coincidence (i.e. \( f_{ro-b} = f_i \)) occurs at the critical cavitation number of \( K_i \approx 12 \), where the value of \( \Delta \phi_{RMS} \) reaches the maximum [45], again
indicating the maximum surge as being a phenomenon of cavitation resonance [46]. The findings from the finest experiments well integrated with computations [47,48], etc., about the part-load, upper-part-load and full-load surges, have provided us with knowledge that is hardly achievable from either analytical or numerical studies alone. The upper-part-load surge that is still quite a controversial subject can be distinguished from the part-load surge by (i) the operating range being 0.7–0.85 times the best efficient discharge rather than 0.2–0.6 times; and (ii) the pulsation frequency being 2–4 times the runner rotational speed rather than 0.2–0.4 times. The upper-part-load pulsations could lead either to a ‘shock phenomenon’ owing to the impact of the vortex rope on the draft-tube wall or the resonance of the harmonics of the vortex rope with the test rig. The work by [47,49] found the pressure–volume relationships for one period $T^*$ of pressure fluctuation to be all in an anticlockwise direction, indicating a circular pattern of providing energy towards the hydraulic system (figure 15).

**Figure 8.** Mean-velocity profile inside the recirculation zone. The dashed line indicates the bubble envelope. The upper half is axial velocity and the lower is azimuthal velocity. The horizontal bars indicate the peak–peak oscillation about the mean. The S denotes the four stagnation points [31].

**Figure 9.** Mean streamline pattern inside the breakdown. The C denotes the centres of recirculation cells [30].

**Figure 10.** Vortex-breakdown position as a function of divergence angle for various Reynolds and circulation numbers. The extension of curves beyond $\gamma = 2.36^\circ$ is valid only for the cases where separation occurs [26].
Velocity distribution at the entrance of the draft-tube is strongly influenced by the flow conditions in the spiral case, distributor and runner, which all change with turbine operating points (head, discharge and cavitation number). For analytical and experimental approaches, it would be impossible to include these influencing factors. Since the 1980s, computational fluid dynamics (CFD) approach has been employed for the studies of draft-tube surges, enabling inclusions of more influencing factors. However, the early calculations are mostly lumped parameter-based models. Nevertheless, they offer clear physical understanding for rather complex problems and are still popular nowadays. The work by Doefler [40] is probably the earliest representative in this category that not only, for the first time, applied the lumped compliance in turbines for counting the elasticity of cavitated draft tube flow, but also further split the cavitation elasticity into a ‘passive’ part and an ‘active’ part. This linearized modelling including the whole hydraulic system of the turbine unit as shown in figure 12 was based on the hydraulic impedance approach. For the lumped elasticity $k$ of cavitation, a passive elasticity $k_{34}$ and an active elasticity $k_{45}$ are defined, respectively, for the axisymmetric part and the spiral rope part (interacting with the elbow and being the exciting source),

$$k_{34} = \frac{dV_{34}}{dH_3} \quad \text{and} \quad k_{45} = \frac{dV_{45}}{dH_k}$$

The pressure surge in the draft tube $\Delta H_R$ is correspondingly split into two parts: the homogeneous element $\Delta H_3$ and the inhomogeneous element $\Delta H_k$, i.e. $\Delta H_R = \Delta H_3 + \Delta H_k$. An estimated ratio $\lambda$ of $k_{45}/k_{34} \approx 0.3$ was assigned to the system with a Francis turbine of specific speed $n_s = 180$. The comparative data for two plants as well as a model have been tested against the calculations, showing great agreement for the critical cavitation number.
The inhomogeneous head $\Delta H_K$ is generated by the inertial and gravitational forces (figure 16),

\[
\begin{align*}
\Delta H_K &= 0.05 \, c_{sr}^3 \quad \text{for inertial force with } c_{sr} = \frac{4Q}{\pi D^2}; \\
\Delta H_K &= 0.3 \, D \quad \text{for gravitational force.}
\end{align*}
\]

The eigenfrequencies of the pipeline ($\omega_p = \omega_{D} / 2L$) and the pure eigenfrequency of the draft-tube ($\omega_c$ for $\Delta Q_3 = 0$), together with the eigenfrequencies of the whole system $\omega_c$ against the cavitational elasticity, have all been obtained as the functions of $k^*$, showing the critical cavitation elasticity $k^*_\text{cr}$ at the intersection of $\omega_c(k^*)$ with the exciting frequency $\omega_{D}$. With the popularity of high-performance cluster workstations in the past two decades, CFD for two-phase unsteady flow in turbine systems is becoming affordable, deepening our understanding of and increasing the scope wider for relevant issues such as surge interacting with the turbine system, governor, power generation and entire electric grid, etc.

By using the concept of flow force [6], a three-dimensional and two-phase cavitated vortex rope model [50] is capable of considering three different vorticity distributions (i.e. Rankine, Scully and Lamb models).

In the study of full-load surge [51], for most cases, the pulsations in different locations of the turbine system are synchronous with the draft-tube surge. So, this type of surge, often referred to as self-excitation, has no direct forces acting upon the system. Instead, the cavitating vortex formed in the draft-tube transfers the hydraulic system into a system without enough positive value of damping factor for stable operations.

3.3. ‘Guide-plate’-induced severe surges

As shown earlier, the surge problem, particularly the low-frequency components created by the draft-tube, is severe and complicated but not well understood yet. Nevertheless, the dimensionless surge frequency $f/\bar{n}$ is linearly and inversely proportional to the specific speed $\bar{n}$, for given head $H$, the rotation speed $\bar{n}$ is inversely proportional to the square root of unit capacity (power $P$). So, the frequency decreases with the increases in specific speed and unit capacity as follows:

\[
\bar{f} = \frac{1}{\bar{n} \cdot \sqrt{P}}.
\]

Therefore, the problem is further worsening nowadays with these compactly designed fast and giant Francis turbines (i.e. higher $\bar{n}$ and $P$-values), because lower frequency makes larger structures (e.g. power plant) and systems (e.g. grid) more prone to oscillations. For the Three Gorges turbines, it lowered to 0.31 Hz the frequency of the strongest amplitude detected on prototype tests in situ [12] but not predicted by the supplier from their model tests or CFDs. This has now been numerically verified by our studies [10,53,54] as 0.336 Hz of the strongest component. Even lower frequencies of components such as 0.15 and 0.265 Hz have also been observed and verified in our studies [10,53,54] as 0.336 Hz of the strongest component. Even lower frequencies of components such as 0.15 and 0.265 Hz have also been measured.

In the context of surges, the guide plate of the Three Gorges turbine was spontaneously spotted in situ as a particular origin generating the low-frequency perturbations...
trapped in the flow entering the turbine system that interacts with draft-tube surges [11]. This could thus be responsible for promoting the boundary-layer streak growth and transition. Therefore, our study was first focusing on the effects of the guide plate. To verify the prediction that this trapped perturbation is responsible for the in situ detected 0.31 Hz strongest component, a CFD simulation for free-stream turbulence was performed. To simulate this large-scale eddy, i.e. the vortex structure around the guide plate that is in the energy-injection range of the turbulence spectrum, the most suitable URANS with RNSG $k-\varepsilon$ model was employed based on the results from our comparison studies on modelling. That is, the chosen model can equally well capture the targeted eddies as LES does but saves a great deal of computational resource by averaging smaller eddies. This allows us to use a small enough grid size and time step for a long enough simulation time. The CFD study employed: 5.84 × 10^5 nodes for whole turbine flow passage, making the wall-adjacent cell’s centroid within the log-law layer $30 < y^+ < 300$; time step within 0.004–0.008 s for achieving the best accurate results; and simulation time more than 180 s for allowing numerical transition to fade out. It has been verified (figure 18) that a highly unsteady and non-axisymmetric ring vortex structure wraps around the guide plate. The pair of vortices vary in time $t$ and azimuth $\theta$. They induce velocity perturbations in the general form of $u' = u(r)e^{i(\sigma t + m\theta + kz)}$ entering the turbine flow system. Here, $u'$ is the velocity perturbation in the cylindrical coordinate $(r, \theta, z)$ with the $\sigma$ being the complex frequency, and $m$ and $k$ being the wavenumbers in angular and axial directions, respectively. It thus enhances the free-stream gust-like pressure fluctuations in the guide-vane passages where their energy is fed into the boundary layer for promoting the streaks there. This high-energy wave also subsequently enters into the draft-tube, promoting draft-tube surges.

To verify the worsening influence of adding the guide plate to overall pressure surges, a part load operating point of guide vane opening $16\%$ and head $H = 67$ m have been found within the main unstable operation zone (280–410 MW). Comparison

---

**Figure 15.** (a) The temporal variations of the vortex rope area and synchronized pressure fluctuations; and (b) the vortex rope area versus the pressure at different locations for one circle of pressure variation $T^*$. $A_w/(A_w + A_b)$ is the ratio of the white area to the total area in the processed image of the rope and is used approximately as the volumetric measure of the rope [47]. (Online version in colour.)

**Figure 16.** Gravitational and inertial forces together constitute the citation pressure zone [40].

The gravitational influence $\Delta H_{\text{geo}}$ and the inertia influence $\Delta H_{\text{inert}} = c_p\frac{c^2 \delta x^2}{2g}$ contribute to the envelope of vortex train position.

**Figure 17.** Self-excitation of pressure pulsations, $f = 3.63$ Hz, in full load [52]. (Online version in colour.)
Since the guide plate has been introduced, the passage frequency of vortex-rope precession (i.e., 0.336 Hz), which approximately coincides with the frequency of the trapped perturbation at the runner and the guide vane opening of 16°, has been increased to 18.75 Hz (figure 19). The amplitudes of the overall surge spectrum are also significantly intensified by the guide plate. In contrast to full load surges, the frequency of this part load surge is thus mainly governed by the vortex-rope precession as confirmed from our studies for the Three Gorges turbine by comparing the precession periods between these two cases for opening 16° (figures 20 and 21). The periods without and with guide plates are approximately 1.8 times different.
and 3.0 s, respectively, which are equivalent to 0.67 and 0.336 Hz. For opening 30° in the upper part load zone and 35° in the full load zone, our studies [10] and others [55] all predicted much lower surges. So far, only for selected representative operating conditions, surge analysis has been performed. Nevertheless, the evidence obtained is convincing as predicted. Further analysis of the surge characteristics such as the phase angle and rotational/synchronous features within the full operation zone is to be performed. Expectedly, these results would be more or less like those reflected in Nishi’s studies mentioned before. As shown in [52], the surge induced at other locations (e.g. the guide vane) could be even higher than those in the draft-tube (figure 17). From a full search of the operation zone, this could also be the case for the Three Gorges puzzle if considering the incoming flow being significantly disturbed by the guide plate immediately upstream of the guide vane.

4. Cavitation triggered by boundary-layer streaks

4.1. Knowledge on cavitation inception in the boundary layer

Cavitation inception triggered by boundary-layer streak breakdown was for the first time conjectured based on the
evidence obtained from the analysis of the Three Gorges puzzle [11]. Indeed, the cavitation inception in the boundary layer was a hot research subject in the last century, in particular from the 1950s to the 1980s, represented by [14,56–58], etc. It was these pioneering studies, plus the recently acquired knowledge of boundary-layer streak transitional processes during the twenty-first century as discussed in §4.2, that inspired the author spontaneously during the in situ inspection. These classic studies explain well the general conditions for nuclei in the boundary layer to cavitate and are overviewed below.

For cavitation inception in the boundary shear layer, the statistical properties of bubbles are dominated by the pressure fluctuation field in terms of their intensity and duration. Therefore, the distribution of the turbulence level, which varies across the boundary layer, alters the statistical characteristics of the microbubbles’ inception performance across the boundary layer. The first study of this phenomenon [56] indicates that the lowest mean pressure across the boundary layer is the location where the turbulence level is highest because

$$\bar{P} + \bar{\nu}^2 = P_1(x). \quad (4.1)$$

Here, $P_1(x)$ is the pressure in the free flow region beyond the boundary layer where $\bar{\nu}^2$ is negligible; and $\bar{P}$ is the mean pressure. It is obvious that bubbles, at this particular location, have the highest probability to cavitate owing to the minimum value of the mean pressure and the maximum value of the instant pressure drop.

The study [57] on the influences of magnitude and time scale of the boundary-layer turbulence on cavitation inception emphasizes the importance of time scale by postulating that the turbulent fluctuations may actually stall the local flow near the wall, leading to a brief period of separation or a turbulent burst with reverse flow. Then, the nuclei within these regions may be exposed to a low pressure longer than would otherwise be the case, thereby promoting the growth of microbubbles. This is evidenced in the experiment of Arakri and Acosta that the frequency of the most unstable T–S wave in the laminar boundary layer just prior to transition is about 5 kHz, equal to a reference time period of 0.2 ms for growth, which is about the same order as the bubble lifetime (0.1 ms) observed in their experiments.

For both the free turbulent shear flow and the fully developed boundary-layer flow, Kolmogorov theory of homogeneous and isotropic turbulence has been employed to relate the properties of the temporal pressure field to cavitation inception. For fully developed boundary layer flow, Arndt & George [14] suggested that if

$$\left(\frac{u'^2}{\nu}\right)_{T_B} < 1 \quad \text{for a smooth wall;} \quad (4.2)$$

$$\left(\frac{u'^2}{\nu}\right)_{T_B} < 1 \quad \text{for a rough wall,}$$

the nuclei in the boundary layer will have enough time to respond to the entire spectrum. Otherwise, only a fraction below the frequency of $1/T_B$ is sensed by the microbubbles. For turbulent boundary-layer flow, the highest frequency in the flow is $(u'^2/\nu)$ for a smooth wall and $(u'^2/h)$ for a rough wall. Thus, if the bubble frequency $1/T_B$ is higher than the highest frequency, the entire spectrum may contribute to bubble growth. For typical nuclei commonly found in water, whose radii are in the range from 100 μm down to 1 μm, the natural frequencies are of the order 5–25 kHz [59]. For a summary of nucleus size distributions in waters, see [60].

Therefore, for the nuclei to cavitate, sufficiently long and sufficiently negative pressure drops generated by boundary-layer flow structures are essential.

4.2. Boundary-layer streak breakdown triggering cavitation inception

We need to explain how the boundary layer could provide such negative pressure drops for the trapped microbubbles (i.e. nuclei) to grow violently during the boundary-layer transition process.

The machines operated mostly at lower head conditions (i.e. at a reduced inlet circulation condition; equation (3.1)), making the pressure surface, instead of the suction surface, of the guide vane (hydrofoil) prone to cavitation. The erosion pattern of narrow and shallow strips suggests a delicate structure of cavitating flow that is very likely to be developed in the boundary layer. The wedged head of strips indicates that the triggering of this cavitation is possibly associated with the formation of turbulent bursts during the boundary-layer transition that is also featured with the wedge head [61] (figure 22). In figure 22c, the breakdown on the centre line and the formation of two regions of highly unsteady flow on either side of the streak are most clearly evident in the contours of the broadband unsteadiness. By comparing this with the typical wedged head in figure 3 and the uncompleted wedged head in figure 23, their resemblance is obvious and the uncompleted widening process seen in figure 23 is thus attributed to the highly random nature of the process in real flows. This uncompleted one-side widening of the wedged head will be further analysed by the concept of GSB developed later. This process indicates that the first breakdown introduces (triggers) a new pair of breakdowns on either side via some instability mechanism, causing span-wise growth of the turbulence production area. This process forms the wedged turbulent burst reflected on the wedged head of the (cavitation) eroded strips.

The analysis [11] on the span-wise stochastic characteristics of the erosion strips was thus focused on the boundary-layer transition and its turbulence production. The boundary-layer transition is full of complicated vortical dynamics such as vortex formation, stretching, pairing and breakdown, etc., though is not fully understood yet. Nevertheless, it was well documented that boundary-layer turbulence plays a significant role in cavitation inception almost half a century ago, first by Daily & Johnson [56] and later by Arndt & George [14], etc. The negative pressure drops generated by these vortical structures are extremely high; for example, the pairing process could produce negative peaks in pressure exceeding the RMS value by a factor of 10 [62]. Therefore, these vortical flow structures (or process) during the boundary-layer transition could easily create such negative pressure drops lasting sufficiently long for nuclei to cavitate. The information about vortical structures involved in the transition process began to be revealed only in the past decade or so from the studies of boundary-layer streaks. Nevertheless, based on the available knowledge as depicted in the following paragraphs, we may say cavitation inception and its evolution in the boundary layer are directly related to the state of the environmental fluids. The rich
vortical flow structures in the boundary layer and their rapid changes in the transitional process could generate local pressure variation capable of triggering cavitation inception if the free-stream (static) pressure there is also sufficiently low (but not necessarily down to the vapour pressure). Note that the negative pressure drop produced by the low-frequency fluctuations in the free stream also contributes to the overall pressure $P_1(x)$ lowering (equation (4.1)) for promoting cavitation susceptibility in the boundary layer.

Early studies [63–67] have already demonstrated span-wise distortion (modulation) induced by various free-stream disturbances. However, it was recent studies, such as [68–75], that have revealed much of the detail of such streak formation, in particular their vortical structures. Among them [75] proposed a well-explained streak formation, revealing detailed vortical dynamics as shown in figure 24. The transitional flow structures consist of the T–S wave and Lambda vortex, the soliton-like coherent structures (SCSs), the secondary closed vortex, the chain of ring vortices and the resultant stream-wise vortices.

Figure 22. True spatial contours in plane, $y = 2$ mm (a) $(\overline{U} + u_\phi)/U_1$; (b) $u_{\phi}'/U_1$. (c) Broadband unsteadiness, $\overline{u}/U_1$ [61]. (Online version in colour.)

Their main features are as follows: (i) the SCSs cause a low-speed streak very close to the wall; (ii) the SCSs have the same frequency as for the T–S wave that is predictable by linear
stability theory; (iii) the vortical dynamics generate significant velocity and pressure fluctuations with \( V_{\text{RMS}} \) being approximately 8–10% of the free stream velocity with the peak value reaching 40%; and (iv) the chain of ring vortices generates velocity fluctuations with a peak value up to 40% at four times the frequency as for the Lambda vortex. For details, see [72,73]. Typical velocity fluctuations are shown by figure 25. All these involve rich vortical dynamics such as the corotating and counterrotating of unequal vortex pairs, etc., in SCS formation. They generate significant negative peaks at frequencies much smaller than those of the bubble nuclei (5–25 kHz).

This streak evolution from the T–S wave can be further visualized by a silt-laden water of concentration 1.5 kg m\(^{-3}\) eroding the guide vane of a Francis turbine\(^5\) as shown in figure 26 [76]. First, the erosion morphology reflects well the vortical structure of the streak. As analysed [78], if the axes of vortices are parallel to the surface, these vortical flow structures will provide significant driving forces making particles impinge on and cut the surface. The vortical structures involved in the streak transitional processes are mostly such cases and therefore leave their footprint well on the damaged surface. Furthermore, the roughened surface and/or the streak-formation process itself could further trigger cavitation owing to abundant nuclei entrained in particles making inception more susceptible, resulting in much intensified silt-cavitation concert erosion [79]. However, for the case of the Three Gorges, the differences are as follows: (i) the damage pattern does not suggest silt erosion; (ii) the span-wise interval of eroded strips is obviously wider than the span-wise wavelength of the boundary-layer streaks as reflected in the silt-erosion case (figure 26); and (iii) the erosion strip does not start immediately after the T–S wave formation, but at a distance further downstream with a wedged head. All these signs suggest that the process of the streak formation immediately after the T–S wave lift-up does not generate appropriate negative peaks for triggering cavitation inception; however, with the further evolution of the streaks under the high receptivity constant\(^6\) to the gust-like perturbations in the free stream until forming a GSB the inception is then triggered. This GSB concept is illustrated in figure 27 by using the streak photo taken from [81]. The first streak will break down randomly in a transitional

Figure 24. Streak structure and breakdown: (a–c) typical hydrogen bubbles in laminar streak formation illuminated by a horizontal laser sheet at different heights [75]; (d) three-dimensional structure reconstructed from these detailed photos [75]. SCS is the soliton-like coherent structure; SV is the secondary closed vortex, \(A\) is the lift-up T–S wave; and 1, 2, 3 and 4 indicate the four ring vortices forming a chain of vortices. Note: the 4 in (d) indicates the front part of the lift-up \(A\) wave that is to form the fourth ring vortex in the chain of vortices through mutual induction with the secondary closed vortex. (Online version in colour.)

Figure 25. Oscilloscope traces of velocity disturbances in both the \(x\)- and \(y\)-directions at various distances from the wall at \(x = 450\) mm. \(T\) represents the period of the T–S waves as 0.5 s [75].
zone as indicated in figure 27; then this breakdown triggers immediately neighbouring streaks, starting breakdowns with tiny time delays, forming the tip of the wedge shape. If these neighbouring streak breakdowns are violent enough to relay the breakdown to further outward streaks on both sides, forming a GSB that produces strong enough turbulence concentrating in a local patch area (often referred to as a local turbulence burst or spot) in the transitional zone with long enough time scale for cavitation to be triggered, then a cavitation inception could occur in that local spot. It is notable that some particular streaks take longer to complete the breakdown process as marked. Some even survived the first triggering from the neighbouring streak breakdown passing through the edge of the GSB but broke down eventually in the second encountered GSB. This could be the influence from the first GSB being too weak and/or the stiffness of the streak itself being strong enough to survive the first GSB. It is also notable that, from roughly every 20 or so of such streaks7 [81], a GSB could form in the transitional zone. This illustration of the GSB from the perspective of the streak transitional process gives a reasonable explanation to a possible cavitation-inception process. For the Three Gorges case, the calculated average span-wise wavelength of such a GSB is \( \lambda_{\text{GSB}} \approx 86 \times 10^{-3} \text{ m} \), approximately equivalent to the observed average erosion-strip spacing \( \lambda_{\text{strip}} \approx 100 \times 10^{-3} \text{ m} \) [11].

The cavitation type triggered through this mechanism could be a foil cavitation such as the one studied by Arakeri82 and commended by Brennen in [59] as being ‘strikingly different’... Sometimes the cavities occur as streaks... Again a transverse periodicity appears to occur...’...Currently there does not appear to be any clear understanding of the reason for the transverse periodicity’. Now, let us revisit this experiment from the perspective of the GSB concept (figure 28). The span-wise wavelength (spacing) of boundary-layer streaks \( \lambda^* \) of this cavitating foil can be estimated as \( \lambda^* = 10\delta^* \) with \( \delta^* \) being the displacement thickness, \( \delta^* = 0.0479 \times 10^{-3} \text{ m} \) (for a typical cavitation streak \( L = 0.012 \text{ m} \) at \( U_0 \approx 15.5 \text{ m s}^{-1} \)). This gives a span-wise wave length of boundary-layer streaks as \( \lambda^* \approx 0.479 \times 10^{-3} \text{ m} \) and an estimated span-wise wavelength of GSB as \( \lambda_{\text{GSB}} \approx 9.58 \text{ mm} \). Again, this value is roughly equivalent to the observed averaged cavitation streak spacing \( \lambda_{\text{streak}} \approx 6.5 \text{ mm} \). Note that the three-dimensional effect disturbed the streak-forming process on both ends of the wing span. Therefore, only five cavitation streaks in the middle span could be used for the estimation of the \( \lambda_{\text{streak}} \) value. Furthermore, the parametric approach used was based on the flat-plate experiment [81]; therefore, this agreement should be considered reasonably well.

This evidence of parametric analysis indicates that the Three Gorges erosion strips are likely to stem from the process of boundary-layer streak formation and breakdown because their span-wise regularities match each other, reflecting well the stochastic characteristics of the turbulence production from the GSBs. However, to make this happen, the free-stream turbulence level, in particular the gust-like component (i.e. the low-frequency pulsations), must be high enough to promote the transition occurring in the FPG zone\(^8 \) of the guide vane. For giant turbines such as the Three Gorges Francis turbine, the huge size of their components and the high free-stream velocity\(^9 \) make the critical Reynolds number\(^{10} \) \( Re_{L, \text{crit}} = U L_{\text{crit}} / \nu \) to be reached in the FPG zone instead of in the APG zone, resulting in higher susceptibility to cavitation inception than in smaller turbines. For the Three Gorges case, the guide plate in the spiral case induces additionally strong fluctuation components of extremely low frequencies (e.g. 0.15 and 0.336 Hz, etc.) coupled with and manifested by the draft-tube surge that significantly promoted this type of cavitation inception.

5. Corrosion appearance

The stainless steel used for the guide vane by nature would not be corroded by the water of the Yangtze River. It is just this type of cavitation that causes the stainless steel to undergo a metallurgical process named ‘sensitization’ and impairs its non-corrosive property.

5.1. Sensitization

Sensitization is a phenomenon of a heat effect well known with austenite stainless steels that causes intergranular corrosion and is a widespread problem. The stainless character of stainless steels occurs when the concentration of chromium exceeds about 12 wt%. When an austenite stainless steel is exposed to a temperature range between 620°C and 676°C, subject to exposure time as well as its own composition, the
carbon will diffuse towards the grain boundaries where the high concentration of carbon ties up the chromium by forming chromium carbides $M_23C_6$ at grain boundaries and leaves a zone of chromium depletion (less than 12%) in the immediate area around the grain boundary. This area thus becomes less corrosion resistant than the bulk material, resulting in intergranular corrosion. For sensitization of austenite stainless steel in turbine corrosion, see [83] and [13 p. 330]. Actually, intergranular corrosion is also a potential problem for martensitic stainless steels, because it is a Fe–Cr–C system and is always used in a tempered condition, in which carbides are precipitated. The investigation [84] shows a sensitized structure detected in martensitic stainless steel (figure 29). The maximum susceptibility to intergranular corrosion has been observed at a tempered condition around $500–550 ^\circ C$ as shown in figure 30.

### 5.2. Heat source

The multi-coloured zones observed (termed as ‘bluing’ in heat treatment) indicate a temperature encountered in the range of $250–600 ^\circ C$. The only possible hydrodynamic mechanism in the turbine capable of generating temperatures at or above this range is cavitation. Single bubble luminescence, e.g. [85–87], has proved a heat source of the order of $10^3 K$.  

![Figure 27](http://rsfs.royalsocietypublishing.org/Interface-Focus/2015/5/10335020)  
**Figure 27.** Illustration shows the process of group-streak breakdown (GSB), leading to the local production of turbulence and randomly forming a local turbulent-burst/spot (enclosure by red dashed line) in the transitional zone with span-wise regularity. The photo of streaks is reproduced from [81]. (Online version in colour.)

![Figure 28](http://rsfs.royalsocietypublishing.org/Interface-Focus/2015/5/10335020)  
**Figure 28.** Streak cavitation on a biconvex hydrofoil at a speed of $15.5 \text{ m s}^{-1}$ and a cavitation number of 0.11 [82]. The flow is from left to right and the leading edge of the foil is about 1 cm from the left-hand edge. Reproduced with permission from V. H. Arakeri. (This photo and caption are cited directly from Brennen [59] and specially acknowledged). (Online version in colour.)

![Figure 29](http://rsfs.royalsocietypublishing.org/Interface-Focus/2015/5/10335020)  
**Figure 29.** Optical micrographs of UNS S41000 steel specimens after an oxalic acid etch test: tempered at $550 ^\circ C$ [84].

![Figure 30](http://rsfs.royalsocietypublishing.org/Interface-Focus/2015/5/10335020)  
**Figure 30.** Comparison among electrochemical tests to discriminate degrees of sensitization for different tempering temperatures. Characteristic values are $i/i_a$ for the DL-EPR, $i_{\text{min}}$ for the potential-static test, and $i_{2\text{max}}/i_{\text{crit}}$ and $Q_{2\text{max}}/i_{\text{crit}}$ for the potential-kinetic polarization. To make different tests comparable, the ratio was plotted between the characteristic value for the material tempered at different temperatures and that for the material tempered at $300 ^\circ C$ [84].
As for hydrodynamically generated bubbles containing mainly water vapour, their collapsing temperatures would be lower [88] than those of gaseous bubbles. The experiment [89] was the first study of hydrodynamic (bubble) cloud luminiscence. Despite the fact that they were not able to provide the temperature, the events of light emission themselves prove that the water molecules are also excited to the temperature of the order of \(10^3\) K and capable of emitting photons. Furthermore, for the Three Gorges case, the bubbles are generated from those boundary-layer streaks very near the boundary surface and therefore collapse much closer to the wall, readily providing a temperature of 500–550°C for sensitization. It is just this metallurgical process that made the martensitic stainless steel corrordible. For details, see [11,90].

6. Similarity laws

The reasons that manufacturers all failed in the first place to detect this problem from their model tests and CFD simulations were as follows. (i) There was absolutely no boundary-layer similarity for model and prototype. With significantly increased size, the prototype is much more susceptible to turbulent transition in the boundary layer owing to its much higher boundary-layer-based Reynolds number. (ii) The free-stream turbulences were also not similar. With the increase of machine size, the draft-tube surges have an overwhelming effect on the free-stream perturbations, in particular for low-frequency components, as analysed before. These two scale effects have been neglected in the hydroturbine industry until the Three Gorges puzzle [11].

6.1. Reynolds number disparity

For the Three Gorges turbines with an extremely large prototype–model ratio of 28, if the similarity of free stream turbulence was required, the Reynolds number equality would lead to \(H_m/H_p = (D_{mp}/D_{1,m})^2\) [11]. That is, the model test could require a test head as high as up to \(H_m = 47,824–88,592\) m. In reality, this is impossible, and the constraint of the test rig resulted in a much smaller Reynolds number for models. For the Three Gorges turbines, the prototype’s Reynolds number is approximately \(Re_m = 4.6 \times 10^7\) at \(Q_{opt} = 718\) m³ s⁻¹, while for models it might be \(10^2–10^3\) smaller.¹²

Dynamic similarity for boundary-layer flows, in particular their characteristics of transitional behaviour, is hardly considered in hydroturbine design. To realize the dynamic similarity of boundary-layer flows, apart from satisfying the free-stream turbulence similarity, the equality of the boundary-layer-based Reynolds number, \(Re_L = UL/\nu\), should also be satisfied. Owing to the constraint condition of the Strouhal number (i.e. \(n_1\) equality), this leads to

\[
\frac{(Re_L)_m}{(Re_L)_p} = \left(\frac{D_{1,m}}{D_{1,p}}\right)^2 \frac{H_m}{H_p}^{1/2} = 1.
\]  

That is, for the case of the Three Gorges (i.e. the prototype–model ratio of 28) the boundary-layer similarity requires

\[
\frac{(Re_L)_m}{(Re_L)_p} = \frac{1}{28} \left(\frac{H_m}{H_p}\right)^{1/2} = 1, \quad \text{i.e.} \quad H_m = 784H_p.
\]  

As mentioned earlier, it is impossible to use such a high head for model tests, leaving the boundary-layer-based Reynolds number for the models far below that for the prototype.

6.2. Geometry disparity (presence of guide plate)

For the machines in the left plant, the flow is further subject to an extra-high level of free-stream turbulence caused by the ‘guide plate’, as shown in figure 31. Obviously, the designer intended to reduce the machine size, \(R_{original}\), without sacrificing cross-sectional area of the spiral case by simply moving the spiral case inwards and leaving the butterfly edge of the stay ring stuck out, renamed as the ‘guide plate’. Indeed, the butterfly edge is designed to be joined with the edge of the spiral case by welding \(in\ situ\) during the installation of large turbines. During the \(in\ situ\) inspection, the author suggested to the Three Gorges authority to remove the guide plate from all the machines in the left plant instead of repairing the torn guide plate and also proposed to reduce the machine size (i.e. the radius \(R_{original}\)) by employing an oval-shaped cross section of the spiral case for future design [11]. This incident explains well that tiny geometrical disparity may result in significant disparity of fluid dynamics including cavitation inception. Our studies have confirmed that this geometrical disparity has caused the strongest component of extremely low frequency 0.336 Hz. Manufacturers are now racing to modify their designs while bidding for new giant hydropower projects.¹³

6.3. Turbulence disparity

Generally speaking, a higher Reynolds number means a higher level of free-stream turbulence. However, it does not necessarily correlate with higher levels of all parameters of free-stream turbulence, in particular for the complex flow systems that involve influences not governed solely by the Reynolds number. Take the influence on turbulence by the cavity in the draft-tube as an example, the cavity volume and shape are also influenced by the Froude number. The Froude number is usually greater for a scaled model, making the cavity more slender (i.e. thinner and longer) and located at a higher elevation. This cavity disparity in turn will change the surge behaviour in a way, as reviewed before, different from what the Reynolds number does. It makes changes of frequencies and amplitudes to the turbulence components mostly in the low-frequency range of the spectrum, whereas those at the high-frequency end could have been suppressed by the cavity compliance. For details of such influence on the surge amplitudes and frequencies of turbines caused by the disparity of the Froude number, readers are
referred to recent studies, e.g. [47]. This influence on the draft-surge similarity was indeed recognized much earlier in 1986 [91] though this is not fully understood yet. By nature, this Froude number determines the pressure gradient relative to the machine size [92] and, therefore, governs the distribution (geometry) of cavitation in the flow. Regarding the influences on surges by the disparity of the cavitation number, it is quite well documented. However, this is usually not the case for model tests because the cavitation number is always kept identical during model tests (see the ‘turbine similarity’ in table 1).

Nevertheless, for the prototype turbines in the left plant of the Three Gorges project, the influences caused by the disparities of both the Reynolds number and the geometry are overwhelming so as to significantly increase the free-stream turbulence level for the low-frequency range. This made them more susceptible to this type of cavitation, whereas their scaled model tests significantly underestimated this risk. Therefore, no cavitation observed during model tests does not guarantee that the Three Gorges turbines in the left plant are free from cavitation of this nature.

7. Conclusion

The modern reaction turbine inherently possesses the problem of draft-tube surges with accompanying cavitation attributed to Euler’s theory-based principles for energy conversion and recovery. The problem is further escalated for giant fast turbines. In such a context, the Three Gorges puzzle has been analysed and a possible hypothesis about its underlying mechanism has been proposed. A revisit of the long-lasting unknown phenomenon observed on the cavitating foil at Caltech further validated this hypothesis.

The identified physical processes behind the puzzle can be summarized as follows. First, the vortex breakdown in swirling flow is one of the mechanisms involved in the draft-tube surge. However, the real surge problem in the turbine system is much more complicated owing to the geometric complexity of the turbine system and the other factors interacting with the surges. The problem becomes worse with the increases in machine unit capacity and specific speed employed. Second, the boundary-layer streak transitional process, which is promoted by the surges, generates strong negative pressure drops at much lower frequencies than the nuclei frequencies. These negative pressure drops superpose on the overall pressure that undergoes periodically pressure troughs of the low-frequency fluctuations in the free stream. This hydrodynamic system provides an ideal environment for nuclei to cavitate on the GSB spots in the FPG zone of guide vanes. Third, the first damaged spot serves well as the nuclei to cavitate on the GSB spots in the FPG zone of the Three Gorges project, the influences caused by the disparities in table 1).

The significance of this puzzle lies in the fact that the Three Gorges turbines developed by all the leading suppliers have all developed more or less the same pattern of erosion although no cavitation was detected during their model tests and CFD simulations. Therefore, it is not an isolated technical problem but reflects a fundamental scientific challenge.

Competing interests. We declare we have no competing interests.

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Endnotes

1The percentage of the remaining kinetic energy in the water leaving the runner is $V_f^2/(2gh) = 0.42(n_{s}/1000)^{1.3}$. For a turbine of very high specific speed, say $n_s=1000$, this could be up to 42%.

11F’ will be used in the article, following the abbreviation employed at the Three Gorges Power Station.

3For the effect of surface roughness on cavitation inception, see [14].

4Leonhard Euler (15 April 1707–18 September 1783) and his first child Johann Albrecht Euler (27 November 1734–17 September 1800).

5For the effect of surface roughness on cavitation inception, see [14].

6For the definition of the receptivity constant of two source perturbations, such as the free-stream vortex and surface vibration (e.g. roughness induced) pertinent to our case, see [80].

7This number is obtained from counting the number of streaks within a typical interval of neighbouring GSBs but is not a statistically averaged value, because there were not enough GSBs in the experiment of [81]. To be accurate, this value should be decided either by experiments or by calculations based on the particular boundary layer being studied. However, this is still beyond reach, which is an area for future studies.

8Where the flow is accelerating, making the overall static pressure lower and more prone to cavitate.

9Of the order of 20–30 m s⁻¹.

10$\approx 3 \times 10^3 \sim 3 \times 10^4$ for a flat plate at a zero angle of attack, subject to a free-stream turbulence level.

11For example, the wall-normal position of the three-dimensional T-surface amplitude maxima is roughly at the non-dimensional distance $y^* = 0.75$ (i.e. $U_f/U_\infty = 0.42$) [80].

12The Reynolds number for the prototype is calculated based on the runner inlet diameter $D_1$ and, for models, it is estimated based on the current conditions of turbine test rigs around the world.

13The manufacturers have all followed the suggestion of modifying their spiral case design such as changing to an oval-shaped cross section, e.g. for Xi Lou Du (溪洛渡), or simply removing the guide plate from the machines later installed in the right plant of the Three Gorges project.
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